Inductors and Inductance

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This lesson provides an introduction to an important electric-circuit element called the inductor and the equally important concept of inductance. When you complete this lesson, you should know the following:

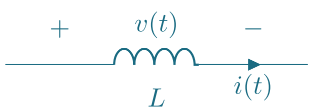
1. How inductors are characterized by a quantity called inductance.
2. The mathematical relationship between the current through and voltage across an inductor.
3. The voltage-current relationship for an inductor in a steady-state sinusoidal (AC) circuit.

Inductors and Inductance

An inductor is a common electric-circuit element that has a very special mathematical relationship between the voltage across and current through its terminals. The schematic symbol we use for an inductor is

where the quantity L is called the *inductance*. The unit for inductance is called the henry, and we denote this unit with the symbol H. If, for instance, an inductor has an inductance equal to 1 mH, then we might represent the inductor in a circuit like this:

If we label the voltage across and current through an inductor like this



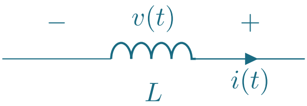
then the voltage and current are related through a mathematical relationship like this:

$$v(t) = L\frac{di(t)}{dt}.$$

That is, the voltage is proportional to the derivative of the current with respect to time, and the constant of proportionality is equal to the inductance. Because of this relationship, the inductance must have units equal to volt-seconds per amp, so a henry is equal to one volt-second divided by one amp:

$$\mathrm{H} = \frac{\mathrm{V} \cdot \mathrm{s}}{\mathrm{A}}.$$

When relating the voltage across to the current through an inductor, it is important to pay careful attention to the voltage polarity and current direction. If, for instance, the voltage and current are labeled like this:



then the voltage and current would be related by the following relationship:

$$v(t) = -L\frac{di(t)}{dt}.$$

If we know the voltage across an inductor, but want to determine the current, then we should use the following relationship:

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau.$$

Inductors in Steady-state AC Circuits

Suppose that the current through an inductor is a steady-state AC function like this:

$$i(t) = A_I \cos(2\pi f t + \theta_I),$$

where f is the frequency (in Hz), A_I is the amplitude (in A) and θ_I is the phase (in radians). In this case, the voltage across the inductor is

$$\begin{aligned}
v(t) &= L\frac{di(t)}{dt} \\
&= L\left[-2\pi f A_I \sin(2\pi f t + \theta_I)\right] \\
&= L\left[-2\pi f A_I \cos(2\pi f t + \theta_I - \pi/2)\right] \\
&= L\left[2\pi f A_I \cos(2\pi f t + \theta_I - \pi/2 + \pi)\right] \\
&= (2\pi f L) A_I \cos(2\pi f t + \theta_I + \pi/2).
\end{aligned}$$

If we represent the current through the inductor by its phasor

$$I = A_I \underline{/\theta_I},$$

then the phasor representation for the voltage across the inductor is

$$\begin{aligned} V &= (2\pi f L) A_I \underline{/\theta_I + \pi/2} \\ &= A_I \underline{/\theta_I} \times (2\pi f L) \underline{/\pi/2}. \end{aligned}$$

Because of this, we associate an inductor with its *steady-state impedance*:

$$\begin{aligned} Z_L &= (2\pi f L) \underline{/\pi/2} \\ &= j(2\pi f L), \end{aligned}$$

so that the stead-state AC relationship between the voltage across and current through an inductor is

$$V = I Z_L.$$

This relationship tells us that the phase of an inductor's impedance is always equal to $90°$, and the magnitude of the impedance is proportional to the frequency. In the special case when the frequency is equal to 0 Hz (DC), the impedance is equal to zero. That is, an inductor behaves like a short when it is in a steady-state DC circuit.