Phasor Representations for Sinusoidal Signals

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This lesson provides an introduction to the use of phasors to represent sinusoidal signals. When you complete this lesson, you should know the following:

1. How to represent a sinusoidal signal as a complex phasor in three different formats: real and imaginary; complex exponential; and amplitude and phase.
2. How to represent a complex phasor with a phase angle specified in either radians or degrees.
3. How to represent a complex phasor for a sinusoidal signal that is defined by either the cosine or sine function.
4. How to represent any phasor so that its phase is in the interval from $-180°$ to $180°$.

Sinusoidal Signals

A sinusoidal voltage or current is of the form:

$$x(t) = A\cos(2\pi f t + \phi),$$

where A is the amplitude (in volts or amperes), f is the frequency (in Hz), and ϕ is the phase (in radians). When all the voltages and currents in a circuit are sinusoidal with the same frequency, we refer to the circuit as a steady-state AC circuit (AC stands for *alternating current*).

The *phasor representation* for a sinusoidal signal is a complex number that is determined by the amplitude and phase of the sinusoid:

$$X = A\cos(\phi) + jA\sin(\phi).$$

The real part of the phasor is the amplitude multiplied by the cosine of the phase, and the imaginary part of the phasor is the amplitude multiplied by sine of the phase. Expressing the phasor representation of the sinusoidal signal in terms of its real and imaginary parts is usually best when we are adding or subtracting two or more sinusoidal signals.

Using Euler's formula we can also represent the phasor X as a complex exponential:

$$X = Ae^{j\phi},$$

or simply as an amplitude and phase:

$$X = A\underline{/\phi}.$$

Each of these representations—real and imaginary, complex exponential, or amplitude and phase—are useful, so it is important that you can easily convert from one form to the other.

Example 1. *Suppose that a voltage signal has an amplitude of* 10 *V, a frequency of* 6 *Hz, and a phase of* $\pi/6$ *radians:*

$$v(t) = 10\cos\left(120\pi t + \frac{\pi}{6}\right).$$

The phasor representation of this signal is:

$$\begin{aligned} V &= 10e^{j\frac{\pi}{6}} \\ &= 10\underline{/\pi/6} \\ &= 8.66 + j5. \end{aligned}$$

Specifying Angles in Degrees

Although the argument of a cosine or sine function must be radians, it is acceptable—and sometimes preferable—to specify the phase of a sinusoidal signal with units of degrees instead of radians:

$$(\text{angle in degrees}) = \frac{180}{\pi} \times (\text{angle in radians}).$$

When we do this, though, we should always be careful to clearly indicate the use of degrees for units by using the $^\circ$ symbol:

$$v(t) = 10\cos\left(120\pi t + 30^\circ\right),$$

or

$$V = 10\underline{/30^\circ}.$$

Range for Angles

Sometimes a sinusoidal signal might be specified with a phase that is greater than 180° or less than -180° (greater than π radians or less than $-\pi$ radians). When that happens, it is good practice to use the relationship that

$$\cos(\theta) = \cos\left(\theta + k360^\circ\right),$$

for any integer k. For example, if a current is specified as

$$i(t) = 5\cos\left(120\pi t + 220^\circ\right),$$

then it is usually good practice to express this current as

$$i(t) = 5\cos\left(120\pi t - 40^\circ\right).$$

Negative Amplitudes

Sometimes a sinusoidal signal might be specified with a leading negative sign like this:

$$i(t) = -5\cos\left(120\pi t + 20^\circ\right).$$

When this happens, we should use the relationship that

$$-\cos(\theta) = \cos\left(\theta \pm 180^\circ\right),$$

so that

$$\begin{aligned} i(t) &= 5\cos\left(120\pi t + 20^\circ \pm 180^\circ\right) \\ &= 5\cos\left(120\pi t - 160^\circ\right). \end{aligned} \tag{1}$$

Here, I chose to subtract 180° rather than add 180° so that the phase would be within the interval between -180° and 180°.

Sinusoids Specified with the Sine Function

Sometimes a sinusoidal signal might be specified with the sine function instead of the cosine function:

$$v(t) = 10\sin\left(2\pi f t - 25^\circ\right).$$

When this happens, we shoud use the relationship that

$$\begin{aligned}\sin(\theta) &= \cos\left(\theta - \frac{\pi}{2}\right)\\ &= \cos\left(\theta - 90^\circ\right).\end{aligned}$$

Therefore, we can write

$$\begin{aligned}v(t) &= 10\sin\left(2\pi f t - 25^\circ\right)\\ &= 10\cos\left(2\pi f t - 25^\circ - 90^\circ\right)\\ &= 10\cos\left(2\pi f t - 115^\circ\right),\end{aligned}$$

or

$$\begin{aligned}V &= 10\underline{/-115^\circ}\\ &= 10\underline{/-0.6389\pi}.\end{aligned}$$

Example 2. *Let's represent the following signal as a phasor with the angle expressed in both radians and degrees:*

$$x(t) = 0.9\cos(2\pi f t + 203^\circ).$$

First, we express the phase in the interval between -180° *and* 180°*:*

$$\begin{aligned}x(t) &= 0.9\cos(2\pi f t + 203^\circ - 360^\circ)\\ &= 0.9\cos(2\pi f t - 157^\circ).\end{aligned}$$

Because -157° *is equal to* 0.872π *radians, the phasor representation for*

$x(t)$ can be written as

$$\begin{aligned} X &= 0.9\underline{/-157^\circ} \\ &= 0.9\underline{/-0.872\pi} \\ &= 0.5788 - j0.6892. \end{aligned}$$

Example 3. *Let's represent the following signal as a phasor with the angle expressed in both radians and degrees:*

$$x(t) = -9.2\sin(120\pi t + 1.7\pi).$$

First, we convert the sine function to a cosine:

$$\begin{aligned} x(t) &= -9.2\sin(120\pi t + 1.7\pi) \\ &= -9.2\cos(120\pi t + 1.7\pi - 0.5\pi) \\ &= -9.2\cos(120\pi t + 1.2\pi). \end{aligned}$$

Next, we remove the leading negative sign:

$$\begin{aligned} x(t) &= 9.2\cos(120\pi t + 1.2\pi + \pi) \\ &= 9.2\cos(120\pi t + 2.2\pi). \end{aligned}$$

Finally, we express the phase in the interval between $-\pi$ and π:

$$x(t) = 9.2\cos(120\pi t + 0.2\pi).$$

Because 0.2π radians is equal to 36°, the phasor representation for $x(t)$ can be written as

$$\begin{aligned} X &= 9.2\underline{/0.2\pi} \\ &= 9.2\underline{/36^\circ} \\ &= 7.443 + j5.408. \end{aligned}$$