Phasor Multiplication and Division

Timothy J. Schulz

This lesson provides an introduction to the multiplication and division of two phasors. When you complete this lesson, you should know the following:

1. How to multiply two phasors.
2. How to divide two phasors.

Multiplication of Phasors

When we multiply two phasors, the result is a phasor whose amplitude is the product of the amplitudes, and whose phase is the sum of the phases. For example, suppose we want to multiply the following phasors:

$$I = 10\underline{/30^\circ},$$

and

$$Z = 2 - j2.$$

To do this, we first note that the phasor Z is equal to

$$Z = \sqrt{8}\underline{/-45^\circ}.$$

The result of the product

$$V = IZ,$$

then, is a phasor whose amplitude is equal to

$$\begin{aligned} \text{amplitude}(V) &= \text{amplitude}(I) \times \text{amplitude}(Z) \\ &= (10)(\sqrt{8}) \\ &= 28.2843, \end{aligned}$$

and whose phase is equal to

$$\begin{aligned} \text{phase}(V) &= \text{phase}(I) + \text{phase}(Z) \\ &= 30^\circ - 45^\circ \\ &= -15^\circ. \end{aligned}$$

Therefore, the product of the two phasors is

$$V = IZ = 28.2843\underline{/-15^\circ}.$$

Multiplication of Phasors

When we divide two phasors, the result is a phasor whose amplitude is the ratio of the amplitudes, and whose phase is the difference of the phases. For example, suppose we want to divide the following phasors:

$$V = 4\angle -90^\circ,$$

and

$$Z = 1 + j\sqrt{3}.$$

To do this, we first note that the phasor Z is equal to

$$Z = 2\angle 60^\circ.$$

The result of the division

$$I = \frac{V}{Z},$$

then, is a phasor whose amplitude is

$$\begin{aligned} \text{amplitude}(I) &= \frac{\text{amplitude}(V)}{\text{amplitude}(Z)} \\ &= \frac{4}{2} \\ &= 2, \end{aligned}$$

and whose phase is equal to

$$\begin{aligned} \text{phase}(I) &= \text{phase}(V) - \text{phase}(Z) \\ &= -90^\circ - 60^\circ \\ &= -150^\circ. \end{aligned}$$

Therefore, the division of the two phasors is

$$I = \frac{V}{Z} = 2\angle -150^\circ.$$