Phasor Addition for Sinusoidal Signals

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This lesson provides an introduction to the use of phasors for the addition of two or more sinusoidal signals with the same frequency. When you complete this lesson, you should know the following:

1. How to add two or more phasors.
2. How to use phasor addition to form the sum of two or more sinusoidal signals that have the same frequency.

Addition of Phasors

When we sum two or more phasors, the result is a phasor whose real part is the sum of the real parts and whose imaginary part is the sum of the imaginary parts. For example, suppose we want to sum the following phasors:

$$V_1 = 10\underline{/45^\circ},$$

$$V_2 = 5\underline{/60^\circ},$$

and

$$V_3 = 8\underline{/-30^\circ}.$$

The result of this summation:

$$V = V_1 + V_2 + V_3,$$

is a phasor whose real part is

$$\begin{aligned} \text{Re}(V) &= \text{Re}(V_1) + \text{Re}(V_2) + \text{Re}(V_3) \\ &= 10\cos(45^\circ) + 5\cos(60^\circ) + 8\cos(-30^\circ) \\ &= 16.4993, \end{aligned}$$

and whose imaginary part is

$$\begin{aligned} \text{Im}(V) &= \text{Im}(V_1) + \text{Im}(V_2) + \text{Im}(V_3) \\ &= 10\sin(45^\circ) + 5\sin(60^\circ) + 8\sin(-30^\circ) \\ &= 7.4012. \end{aligned}$$

Therefore, the phasor sum is

$$\begin{aligned} V &= \text{Re}(V) + j\text{Im}(V) \\ &= 16.4993 + j7.4012 \\ &= 18.08\underline{/24.16^\circ}. \end{aligned}$$

Addition of Sinusoidal Signals

The addition of two sinusoidal signals with the same frequency is easily accomplished by using phasor addition. That is, the phasor representation for the signal $v(t)$, where

$$v(t) = v_1(t) + v_2(t) + v_3(t),$$

when the signals $v_1(t)$, $v_2(t)$, and $v_3(t)$ are sinusoids with the same frequency is the sum of the individual signal phasors:

$$V = V_1 + V_2 + V_3.$$

For instance, suppose that we want to evaluate the summation of the following signals:

$$v_1(t) = 4\cos(120\pi t + 50^\circ),$$

$$v_2(t) = -10\cos(120\pi t - \pi/3),$$

and

$$v_3(t) = 6\sin(120\pi t - 100^\circ).$$

To evaluate this summation, we first convert each signal to its phasor representation:

$$V_1 = 4\underline{/50^\circ},$$

$$V_2 = 10\underline{/-\pi/3 + \pi} = 10\underline{/2\pi/3} = 10\underline{/120^\circ},$$

and

$$V_3 = 6\underline{/-100^\circ - 90^\circ} = 6\underline{/-190^\circ} = 6\underline{/170^\circ}.$$

The real part of the phasor sum, then, is equal to

$$\begin{aligned}\text{Re}(V) &= \text{Re}(V_1) + \text{Re}(V_2) + \text{Re}(V_3) \\ &= 4\cos(50^\circ) + 10\cos(120^\circ) + 6\cos(170^\circ) \\ &= -8.3377,\end{aligned}$$

and the imaginary part is equal to

$$\begin{aligned}\text{Im}(V) &= \text{Im}(V_1) + \text{Im}(V_2) + \text{Im}(V_3) \\ &= 4\sin(50^\circ) + 10\sin(120^\circ) + 6\sin(170^\circ) \\ &= 12.7663.\end{aligned}$$

The phasor for the signal $v(t)$, then, is equal to

$$\begin{aligned}V &= -8.3377 + j12.7663 \\ &= 12.2478\underline{/123.15^\circ},\end{aligned}$$

and the corresponding sinusoidal signal is

$$v(t) = 12.2478\cos(120\pi t + 123.15^\circ).$$