General Ad-hoc Methods

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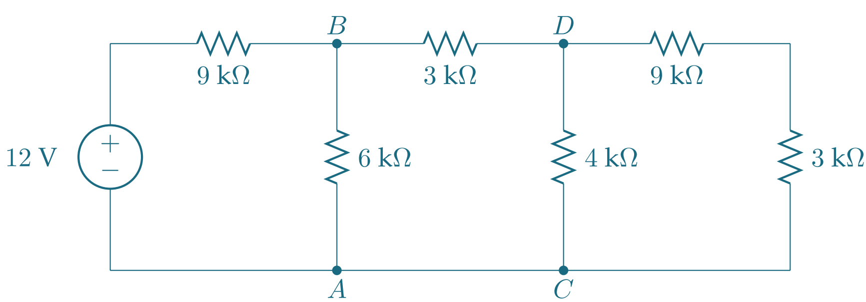
This lesson provides an overview of the ad-hoc application of Ohm's Law, Kirchhoff's Current and Voltage Laws, and the concept of equivalent resistance for series and parallel resistors to solve for unknown currents and voltages in resistive circuits. When you complete this lesson, you should know the following:

1. How to simplify a circuit by applying the concept of equivalent resistance.
2. How to use Ohm's Law and Kirchhoff's Laws to determine the voltages and currents in a simplified circuit.
3. How to expand a simplified circuit back to its unsimplified form, and to use voltage and current division to solve for voltages and currents.

In the following sections we'll use a few example circuits to illustrate the general principles that are used to employ ad-hoc methods to solve for voltage and currents in circuits.

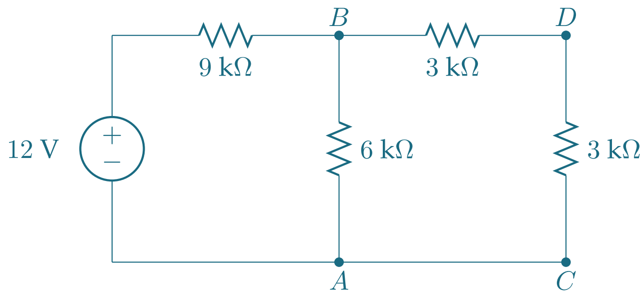
Example 1

Let's begin with the circuit shown below:



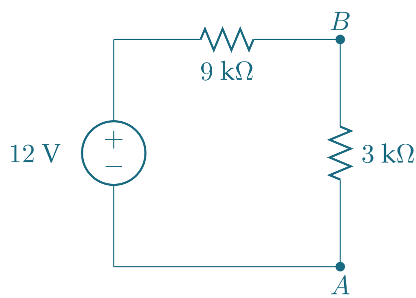
To solve for the voltages and currents in this circuit, let's first combine the $9\ \text{k}\Omega$ and $3\ \text{k}\Omega$ resistors on the far right of the circuit as a single $12\ \text{k}\Omega$ resistance, and then note that this resistance is in parallel with the $4\ \text{k}\Omega$ resistor. Then we can replace those three resistors between nodes C and D with a single equivalent resistance with the value

$$R_{CD} = \frac{(12)(4)}{12+4} = \frac{48}{16} = 3\ \text{k}\Omega.$$



Next, we note that the two $3\ \mathrm{k}\Omega$ resistors can be combined in series as a single $6\ \mathrm{k}\Omega$ resistance, then that resistance can be combined in parallel with the $6\ \mathrm{k}\Omega$ resistor to provide an equivalent resistance between nodes A and B with the value:

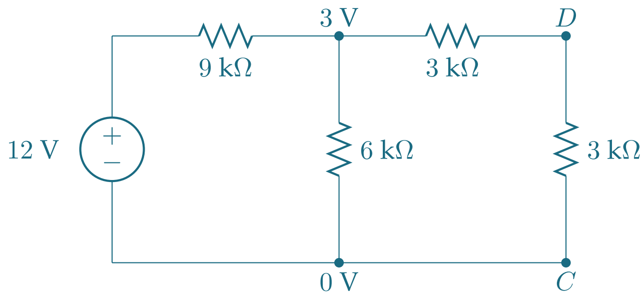
$$R_{AB} = \frac{(6)(6)}{6+6} = \frac{36}{12} = 3\ \mathrm{k}\Omega.$$



Now, if we select node A as our reference and assign to it a voltage of 0 V, then we can use voltage division to solve for the voltage at node B:

$$V_B = 12\frac{3}{9+3} = \frac{36}{12} = 3\ \mathrm{V}.$$

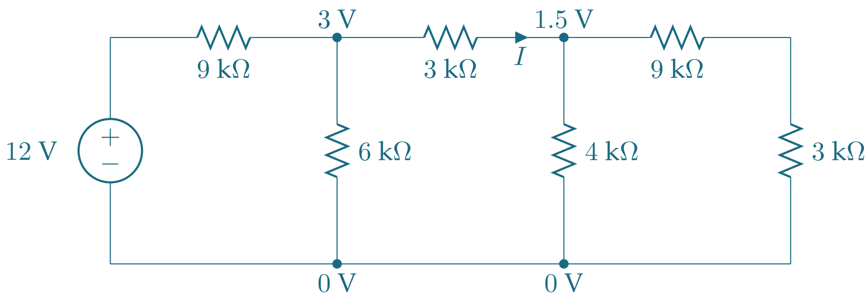
Next, using this value for the voltage at node B we can use voltage division to solve for the voltage at node D in the expanded circuit:



That is,

$$V_D = 3\frac{3}{3+3} = \frac{9}{6} = 1.5\ \text{V},$$

and, of course, $V_C = 0\ \text{V}$. Finally, we can expand back to the original circuit and use the voltages at nodes A, B, C and D to solve for any voltage or current in the circuit.

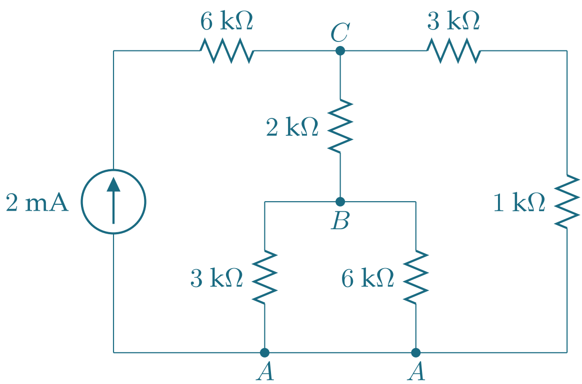


The current flowing left to right through the $3\ \text{k}\Omega$ resistor at the top of the circuit, for instance, is evaluated as:

$$I = \frac{3 - 1.5}{3000} = 0.5\ \text{mA}.$$

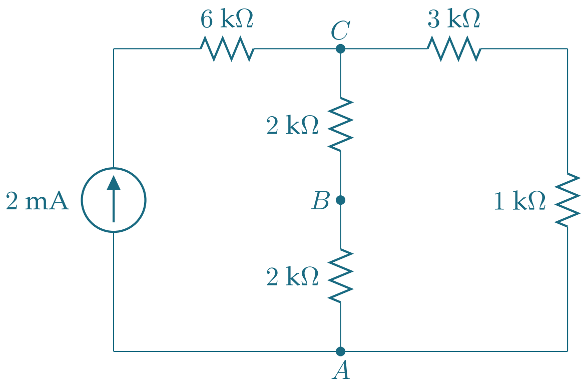
Example 2

For another example of the application of ad-hoc methods, let's take a look at the following example circuit:

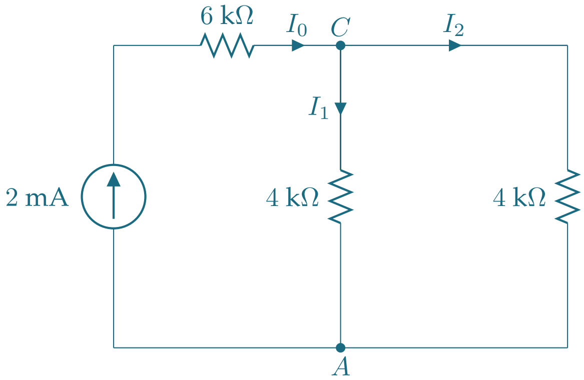


For this circuit, we've labeled both nodes at the bottom of the circuit as A because they are connected by a simple wire and are, effectively, the same node. As a first step in analyzing this circuit, we might combine the two resistors between nodes A and B as a single resistance with the value:

$$R_{AB} = \frac{(3)(6)}{3+6} = \frac{18}{9} = 2\ \text{k}\Omega.$$



Next, we can combine the 2 kΩ and 2 kΩ resistors in series, and the 3 kΩ and 1 kΩ resistors in series to further simplify the circuit.



Now we note that the current labeled I_0 is provided by the current source and has the value $I_0 = 2$ mA. Then, by using the current division method, we can solve for the currents I_1 and I_2:

$$I_1 = \frac{4}{4+4} I_0 = \left(\frac{1}{2}\right) 2 \text{ mA} = 1 \text{ mA},$$

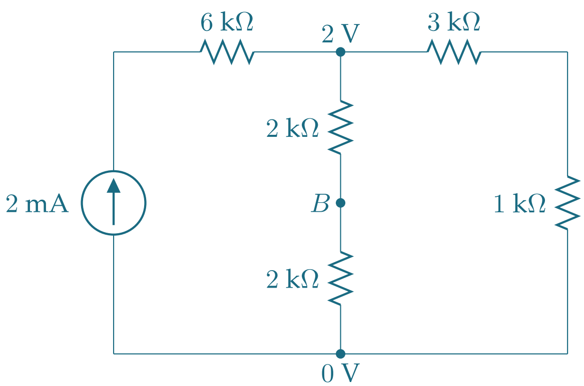
and

$$I_2 = \frac{4}{4+4} I_0 = \left(\frac{1}{2}\right) 2 \text{ mA} = 1 \text{ mA},$$

If we set node A as our reference and define its voltage as $0V$, then the voltage at node C is evaluated as

$$V_C = I_1(4\,\text{k}\Omega) = 4\text{ V}.$$

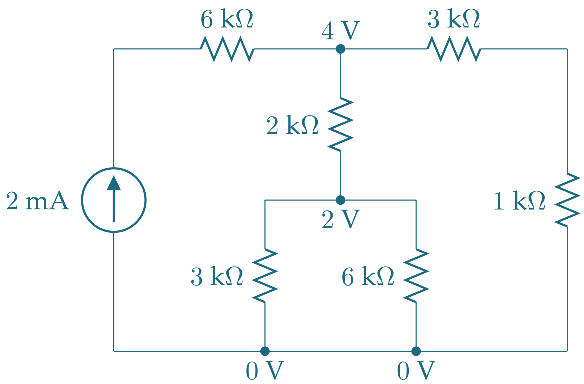
Then, we can reinsert node B into the circuit:



and use voltage division to solve for its voltage:

$$V_B = \frac{2}{2+2}V_C = \left(\frac{1}{2}\right) 4\text{ V} = 2\text{ V}.$$

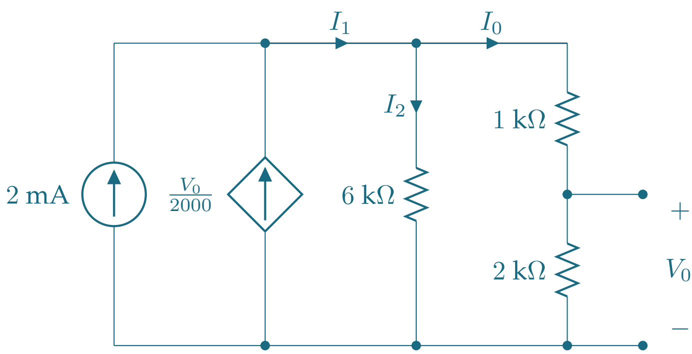
Finally, we can redraw the original circuit with all the node voltage values identified:



Now, with these voltages identified, we can easily solve for any voltage, current, or power in this circuit.

Example 3

For our final example, let's find the voltage labeled V_0 in the following circuit:



To solve for V_0, we first use the current division method to note that the current labeled I_0 is related to the current labeled I_1 according to

$$I_0 = \frac{6}{6+3} I_1 = \frac{2}{3} I_1.$$

Then, we note that the voltage labeled V_0 is related to this current according to

$$V_0 = 2000 I_0 = \frac{4000}{3} I_1.$$

Next, we use Kirchhoff's current law to get the following expression for the current I_1:

$$I_1 = 0.002 + \frac{V_0}{2000}.$$

Putting these two equations together we can solve for the voltage V_0:

$$V_0 = \frac{4000}{3}\left(0.002 + \frac{V_0}{2000}\right)$$

or

$$V_0 = 8 \text{ V}.$$