Node-Voltage Method: Part 3

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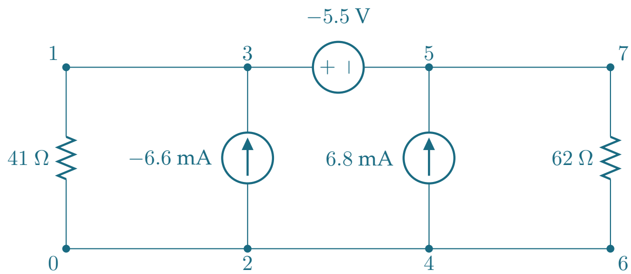
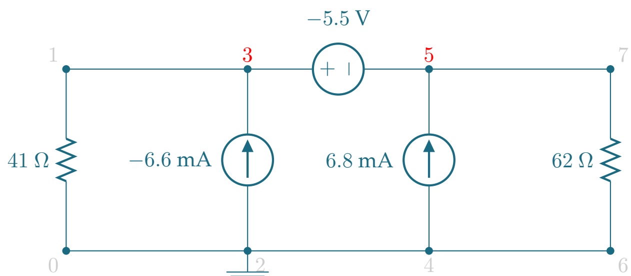
This lesson provides guidance on how to apply the node-voltage method to circuits that contain one or more voltage sources. When you complete this lesson, you should know the following:

1. How to write and solve node-voltage equations for a circuit that contains voltage sources between any of its nodes.

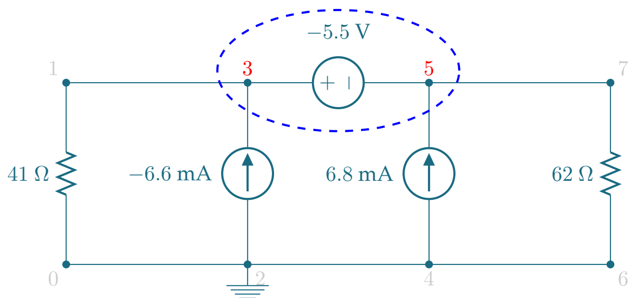
We've already looked at circuits that contain a voltage source connected to the reference (or ground) node. In those situations, the voltage source simply establishes the node voltage for the node to which it is connected. Now we'll look at situations for which the voltage source is connected to nodes other than the reference node. To demonstrate how we utilize the node-voltage method for these situations, let's begin with the following circuit:



Although we've assigned node labels for all the nodes, we should note that nodes 1 and 2 are the same, as are nodes 5 and 7, as are nodes 0, 2, 4 and 6. Therefore, if we define node 2 as our reference, then we can determine all the voltages in this circuit by solving for the voltages at nodes 3 and 5, which are shown in red in the following circuit diagram:



Now, if we begin by applying Kirchhoff's Current Law (KCL) at node 3, we could use Ohm's law to determine the current flowing through the $41\ \Omega$ resistor ($V_3/41$), and we could use the definition of a current source to determine the current flowing through the -6.6 mA source. We can not, however, write a simple expression to describe the current flowing through the -5.5 V voltage source. Because of this, we cannot use KCL to write an equation for the currents flowing from node 3. Likewise—and for the same reason—we cannot write an equation for the currents flowing out of node 5. To address this issue, we make use of the concept of a *super-node* by grouping the nodes connected by the voltage source:



If we note that the voltages at nodes 3 and 5 are related by the following equation:

$$V_5 = V_3 + 5.5,$$

then we can apply KCL to the super-node contained within the blue ellipse to obtain the following equation:

$$\frac{V_3}{41} - \frac{-6.6}{1000} - \frac{6.8}{1000} + \frac{V_3 + 5.5}{62} = 0,$$

or

$$\left(\frac{1}{41} + \frac{1}{62}\right) V_3 = \frac{6.8 - 6.6}{1000} - \frac{5.5}{62}.$$

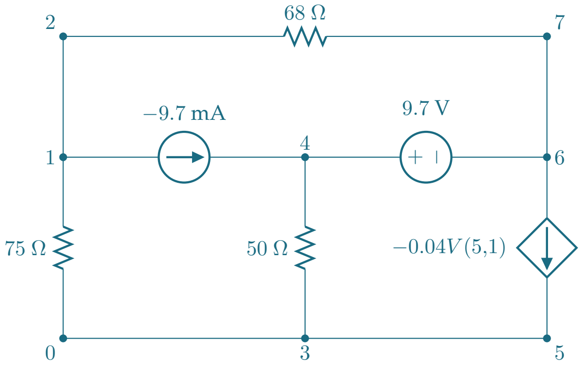
Therefore, we can determine that

$$V_3 = -2.18\ \text{V},$$

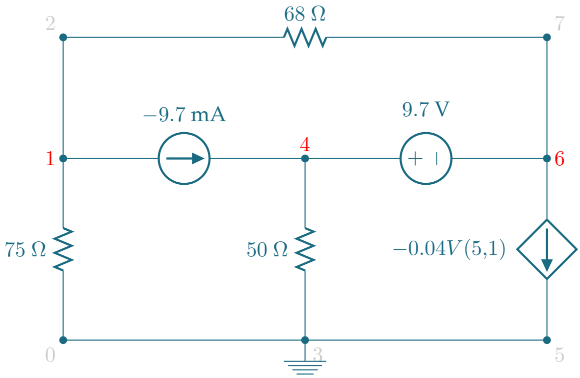
and

$$V_5 = 3.32 \text{ V}.$$

Now let's look at another example:



As with previous examples, we've labeled all of the circuit's nodes, but if we select nodes 0, 3 and 5 as the reference (or ground), then we can solve for all of the voltages in this circuit by finding the voltages at nodes 1, 4 and 6:



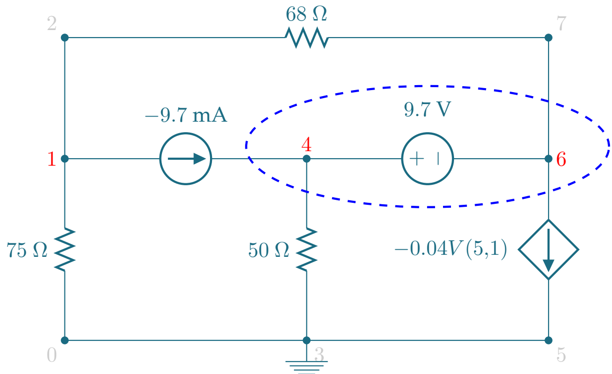
The dependent current source between nodes 6 and 5 is controlled by the voltage drop from node 5 to node 1, but because node 5 is our reference, we can write

$$V(5,1) = V_5 - V_1 = 0 - V_1 = -V_1.$$

If we begin at node 1, our first equation is

$$\frac{V_1 - V_6}{68} + \frac{-9.7}{1000} + \frac{V_1}{75} = 0. \qquad \text{(Node 1)}$$

Next, if we attempt to write an equation for node 4, we will encounter a problem with the voltage source between nodes 4 and 6. To deal with this, we combine nodes 4 and 6 as a *super-node*,



and then apply KCL to that region, noting that $V_4 = V_6 + 9.7$:

$$\frac{9.7}{1000} + \frac{V_6 - V_1}{68} - 0.04(0 - V_1) + \frac{V_6 + 9.7}{50} = 0. \qquad \text{(Super-node)}$$

Grouping terms in these two equations, we get the following system:

$$\left(\frac{1}{68} + \frac{1}{75}\right) V_1 - \frac{1}{68} V_6 = \frac{9.7}{1000}, \qquad \text{(Node 1)}$$

and

$$-\left(0.04+\frac{1}{68}\right)V_1+\left(\frac{1}{68}+\frac{1}{50}\right)V_6=-\left(\frac{9.7}{1000}+\frac{9.7}{50}\right). \tag{Super-node}$$

Then, the following code could be used in an interactive Matlab session to solve for the unknown node voltages:

```
>> A = [(1/68 + 1/75) -1/68; ...
       -(0.04 + 1/68) (1/68 + 1/50)];
>> b = [9.7/1000; -(9.7/1000 + 9.7/50)];
>> v = inv(A)*b

v =

  -15.7681
  -30.7242
```

Therefore, we would conclude that

$$V_1=-15.77\text{ V},$$

$$V_6=-30.72\text{ V},$$

and

$$V_4=V_6+9.7=-21.02\text{ V}.$$