Node-Voltage Method: Part 2

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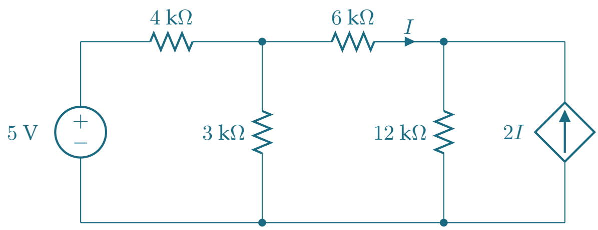
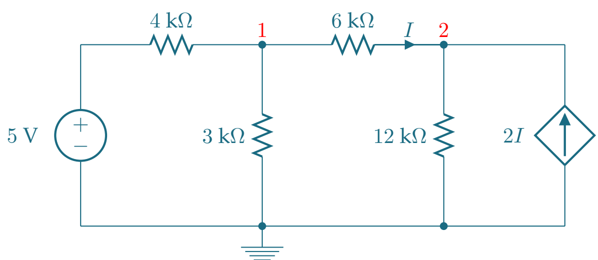
This lesson provides guidance on how to apply the node-voltage method to circuits that contain one or more controlled sources. When you complete this lesson, you should know the following:

1. How to write and solve node-voltage equations for a circuit that contains both independent and dependent voltage or current sources.

To demonstrate how we utilize the node-voltage method for analyzing a circuit with a dependent source, let's begin with the following circuit:



For this circuit, the dependent source provides a current with a value that is equal to twice the value of the current flowing from left to right through the 6 kΩ resistor. To apply the node-voltage method for this circuit, we first identify a reference node and label the remaining nodes for the circuit:



Next, we begin at node 1 and set the sum of all the currents flowing out of the node to zero:

$$\frac{V_1 - 5}{4000} + \frac{V_1 - V_2}{6000} + \frac{V_1}{3000} = 0. \qquad \text{(Node 1)}$$

Then we proceed to node 2 and set the sum of all the currents flowing from that node to zero:

$$\frac{V_2 - V_1}{6000} - 2I + \frac{V_2}{12000} = 0. \qquad \text{(Node 2)}$$

At this point we have two equations but three unknowns (V_1, V_2, and I), so we need another equation. We can easily resolve this issue by simply writing another equation for the current I:

$$I = \frac{V_1 - V_2}{6000}. \qquad \text{(Dependent Source Current)}$$

Now we can substitute this relationship into the equation for Node 2:

$$\frac{V_2 - V_1}{6000} - 2\frac{V_1 - V_2}{6000} + \frac{V_2}{12000} = 0, \qquad \text{(Node 2)}$$

which gives us the following two equations for the two unknown node voltages:

$$\frac{9}{12000}V_1 - \frac{1}{6000}V_2 = \frac{5}{4000}, \qquad \text{(Node 1)}$$

and

$$\frac{-3}{6000}V_1 + \frac{7}{12000}V_2 = 0. \qquad \text{(Node 2)}$$

If we multiply each of these equations by 12000 we can rewrite them as:

$$9V_1 - 2V_2 = 15, \qquad \text{(Node 1)}$$

$$-6V_1 + 7V_2 = 0, \qquad \text{(Node 2)}$$

and we can express this system of equations in matrix-vector form as:

$$\begin{bmatrix} 9 & -2 \\ -6 & 7 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \end{bmatrix}.$$

At this point we can use Cramer's rule, a programmable calculator, or a programming language like Matlab to solve for the unknown node voltages. Here, for instance, is an example of how an interactive session in Matlab might be used to solve for the voltages:

```
>> A = [9 -2; -6 7];
>> b = [15; 0];
>> v = inv(A)*b

v =

    2.0588
    1.7647
```

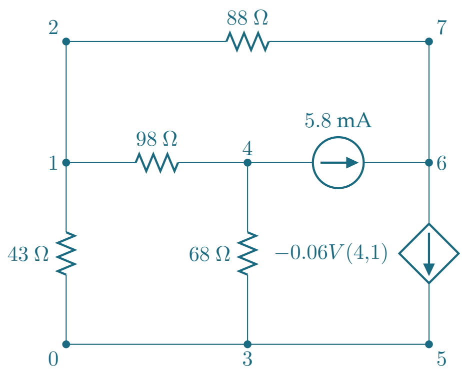
Therefore, we would conclude that

$$V_1 = 2.0588 \text{ V},$$

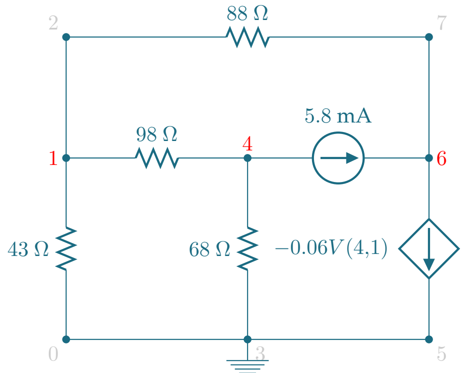
and

$$V_2 = 1.7647 \text{ V}.$$

Now, let's take a look at another example circuit:



For this circuit, we've labeled all the nodes with a numbers 0 through 7, and the dependent current source is controlled by the voltage *drop* from node 4 to node 1. That is, the current flowing from node 6 toward node 5 is equal to -0.06 times the voltage drop from node 4 to node 1. Although we have assigned labels to all the nodes, we should note that nodes 1 and 2 are the same, as are nodes 6 and 7, as are nodes 0, 3 and 5. If we assign node 3 as our ground, then we can solve for all the voltages in this circuit by finding the voltages at nodes 1, 4 and 6, which are shown in red in the following circuit diagram:



Beginning with node 1, our first equation is

$$\frac{V_1 - V_6}{88} + \frac{V_1 - V_4}{98} + \frac{V_1}{43} = 0. \qquad \text{(Node 1)}$$

Next, the equation for node 4 is

$$\frac{V_4 - V_1}{98} + \frac{5.8}{1000} + \frac{V_4}{68} = 0, \qquad \text{(Node 4)}$$

and the equation for node 6 is

$$\frac{-5.8}{1000} + \frac{V_6 - V_1}{88} - 0.06V(4,1) = 0. \qquad \text{(Node 6)}$$

Because $V(4,1) = V_4 - V_1$, we can rewrite this last equation as

$$\frac{-5.8}{1000} + \frac{V_6 - V_1}{88} - 0.06(V_4 - V_1) = 0. \qquad \text{(Node 6)}$$

Now we can group the common terms in these equations to get:

$$\left(\frac{1}{88}+\frac{1}{98}+\frac{1}{43}\right)V_1-\frac{1}{98}V_4-\frac{1}{88}V_6=0, \qquad \text{(Node 1)}$$

$$\frac{-1}{98}V_1+\left(\frac{1}{98}+\frac{1}{68}\right)V_4+0V_6=\frac{-5.8}{1000}, \qquad \text{(Node 4)}$$

and

$$\left(0.06+\frac{1}{88}\right)V_1-0.06V_4+\frac{1}{88}V_6=\frac{5.8}{1000}. \qquad \text{(Node 6)}$$

Finally, we can express this system of equations in matrix-vector form:

$$\begin{bmatrix} \left(\frac{1}{88}+\frac{1}{98}+\frac{1}{43}\right) & -\frac{1}{98} & -\frac{1}{88} \\ -\frac{1}{98} & \left(\frac{1}{98}+\frac{1}{68}\right) & 0 \\ \left(0.06+\frac{1}{88}\right) & -0.06 & \frac{1}{88} \end{bmatrix} \begin{bmatrix} V_1 \\ V_4 \\ V_6 \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{5.8}{1000} \\ \frac{5.8}{1000} \end{bmatrix},$$

and, using an interactive session in Matlab, we can solve for the unknown voltages:

```
>> A = [(1/88 + 1/98 + 1/43) -1/98 -1/88; ...
        -1/98 (1/98 + 1/68) 0; ...
        (0.06-1/88) -0.06 1/88];
>> b = [0; -5.8/1000; 5.8/1000];
>> v = inv(A)*b

v =

   -0.1630
   -0.2996
   -0.3739
```

Therefore, we would conclude that

$$V_1 = -163.0\,\text{mV},$$

$$V_4 = -299.6\,\text{mV},$$

and

$$V_6 = -373.9\,\mathrm{mV}.$$