Mesh-Current Method: Part 3

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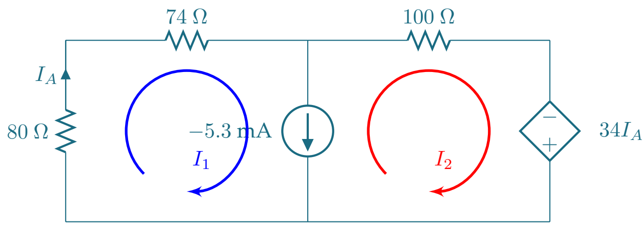
This lesson provides guidance on how to apply the mesh-current method to circuits that contain one or more dependent sources. When you complete this lesson, you should know the following:

1. How to write and solve mesh-current equations for a circuit that contains one or more dependent sources in any of its loops.

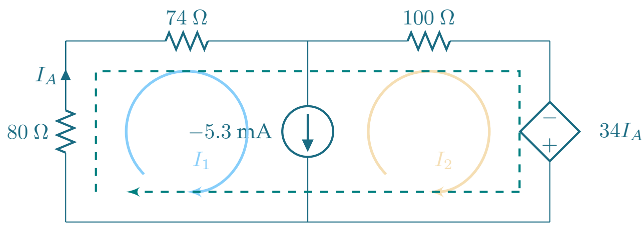
Let's begin by considering the circuit shown below with two mesh currents labeled I_1 and I_2:



Because the current source is shared by the two mesh currents, we can begin with the the source constraint equation:

$$I_1 - I_2 = -5.3 \text{ mA}. \qquad \text{(Source Constraint)}$$

Then, using the super loop shown below:



and noting that the dependent source is controlled by the upward-flowing current through the 80 Ω resistor, which is equal to the mesh current I_1, we can obtain another equation for the two mesh currents:

$$80I_1 + 74I_1 + 100I_2 - 34I_1 = 0. \qquad \text{(Super Loop)}$$

Grouping common terms in these two equations provides the following system of equations:

$$I_1 - I_2 = \frac{-5.3}{1000}, \qquad \text{(Source Constraint)}$$

and

$$120I_1 + 100I_2 = 0. \qquad \text{(Super Loop)}$$

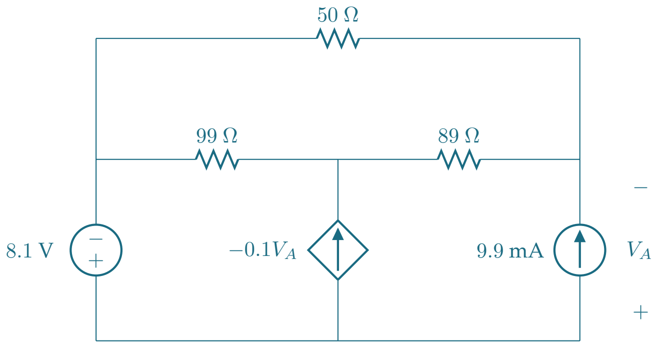
Solving this system of equations results in

$$I_1 = -2.4\,\text{mA},$$

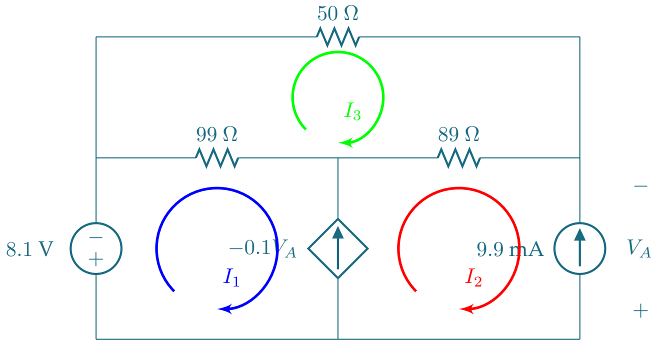
and

$$I_2 = 2.9\,\text{mA}.$$

Next, let's consider the following circuit:



This circuit contains an independent current source in one of the loops, and a dependent current source that will appear in two of the loops. The dependent source is controlled by the voltage drop (in the upward direction) across the independent current source. To analyze this circuit, we can begin by defining the three mesh currents as shown below:



First, we can use the independent source to obtain an equation for one of the mesh currents:

$$I_2 = -9.9 \text{ mA}. \qquad \text{(Source Constraint)}$$

Then we can apply KVL around the outer loop of the circuit to obtain an expression for the controlling voltage (V_A) in terms of the mesh currents:

$$V_A = 8.1 + 50I_3.$$

Next, we can use the controlled source to obtain a relationship between the mesh currents I_1 and I_2:

$$\begin{aligned} I_2 - I_1 &= -0.1V_A \\ &= -0.1(8.1 + 50I_3), \end{aligned}$$

or

$$I_2 - I_1 + 5I_3 = -0.81. \qquad \text{(Dependent Source Contraint)}$$

Finally, we can write an equation around the I_3 loop:

$$50I_3 + 89(I_3 - I_2) + 99(I_3 - I_1) = 0. \qquad \text{(Loop 3)}$$

Then we can regroup the Source Constraint, Dependent Source Constraint, and Loop 3 equations to obtain the following three equations:

$$0I_1 + I_2 + 0I_3 = -9.9 \times 10^{-3},$$

$$-I_1 + I_2 + 5I_3 = -0.81,$$

and

$$-99I_1 - 89I_2 + 138I_3 = 0,$$

which result in

$$I_1 = -723.81 \text{ mA},$$

$$I_2 = -9.9 \text{ mA},$$

and

$$I_3 = -304.78 \text{ mA}.$$