Mesh-Current Method: Part 2

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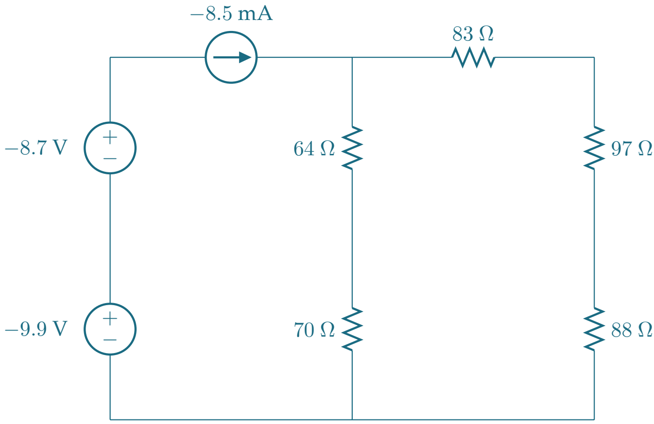
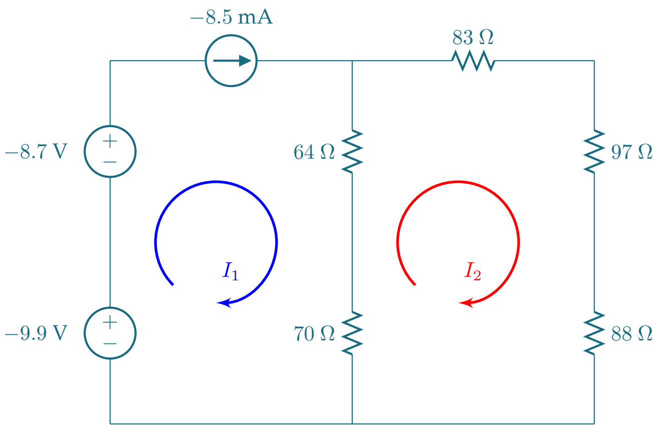
This lesson provides guidance on how to apply the mesh-current method to circuits that contain one or more current sources. When you complete this lesson, you should know the following:

1. How to write and solve mesh-current equations for a circuit that contains current sources in any of its loops.

Let's begin by considering the following circuit:



If we define the loops and associated mesh currents as shown below,



then we might begin by applying Kirchhoff's Voltage Law (KVL) to the loop associated with mesh current I_1. As we've done with previous applications of the mesh-current method, the voltage drops associated with the voltage sources could be determined by the source values, and the voltage drops associated with the resistors could be related to the mesh currents through Ohm's Law. We do not, however, have a method for associating a voltage drop with the -8.5 mA source. Rather than causing a problem, though, the presence of the current source greatly simplifies our analysis by directly specifying the value for the mesh current I_1. That is, the presence of the current source requires that

$$I_1 = -8.5 \text{ mA}. \qquad \text{(Source Constraint)}$$

This simplification always happens in this way when a current source appears in only one loop. Now, we can apply KVL to loop 2:

$$70(I_2 - I_1) + 64(I_2 - I_1) + 83I_2 + 97I_2 + 88I_2 = 0, \qquad \text{(Loop 2)}$$

and these two equations allow us to solve for the two mesh currents:

$$I_1 = -8.5 \text{ mA},$$

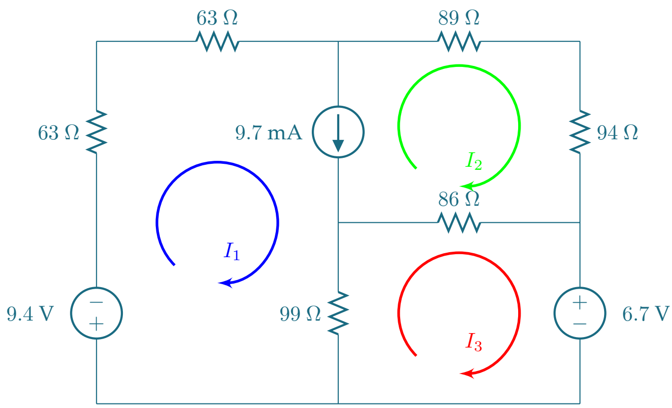
and

$$402I_2 = 134I_1 \Longrightarrow I_2 = \frac{1}{3}I_1 \simeq -2.83 \text{ mA}.$$

Then, for example, if we need to know the voltage drop across the $64\ \Omega$ resistor in the downward direction we could compute it as follows:

$$V_{64\ \Omega} = 64(I_1 - I_2) = 64(I_1 - I_1/3) = 64(2/3)I_1 = -362.7 \text{ mV}.$$

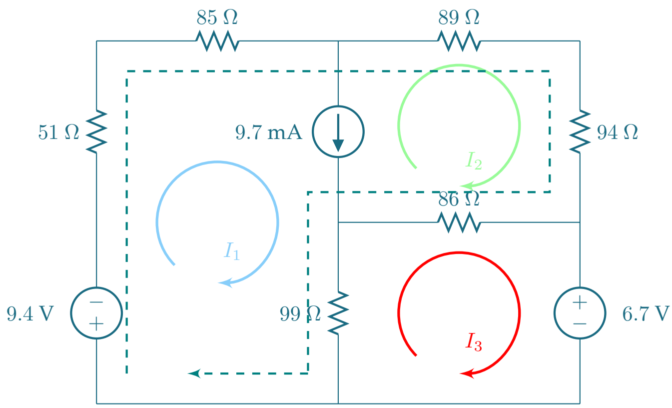
Now let's look at an example for which a current source is in a path that is shared by two loops:



If we apply KVL to either loop 1 or loop 2, we will not be able to easily specify the voltage drop associated with the current source that is shared by the two loops. To deal with this, we first note that the source specifies a relationship between the two mesh currents:

$$I_1 - I_2 = 9.7\ \mathrm{mA}. \qquad \text{(Source Constraint)}$$

This relationship provides one equation for the two mesh currents. To specify another equation, we utilize a *super-loop* that combines the two loops, as shown with the dashed line below:



Applying KVL to this loop, we obtain the following equation:

$$9.4 + 51I_1 + 85I_1 + 89I_2 + 94I_2 + 86(I_2 - I_3) + 99(I_1 - I_3) = 0, \quad \text{(Super Loop)}$$

and applying KVL to loop 3 provides:

$$99(I_3 - I_1) + 86(I_3 - I_2) + 6.7 = 0. \quad \text{(Loop 3)}$$

After we group the common terms, these three equations provide the following system of equations:

$$I_1 - I_2 + 0I_3 = \frac{9.7}{1000}, \quad \text{(Source Constrain)}$$

$$235I_1 + 269I_2 - 185I_3 = -9.4, \quad \text{(Super Loop)}$$

and

$$-99I_1 - 86I_2 + 185I_3 = -6.7. \quad \text{(Loop 3)}$$

Then, solving these three equations for the unknown mesh currents results in

$$I_1 = -44.9\,\text{mA},$$

$$I_2 = -54.6 \text{ mA},$$

and

$$I_3 = -85.6 \text{ mA}.$$

If, then, we need to know the current flowing downward through the $99\ \Omega$ resistor, we could easily compute that as $I_1 - I_3$.