Mesh-Current Method: Part 1

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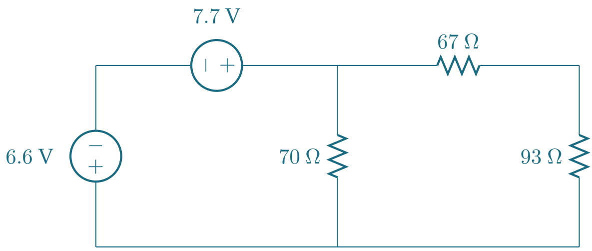
This lesson provides an introduction to the mesh-current method as it is commonly used in the study of electric circuits. When you complete this lesson, you should know the following:

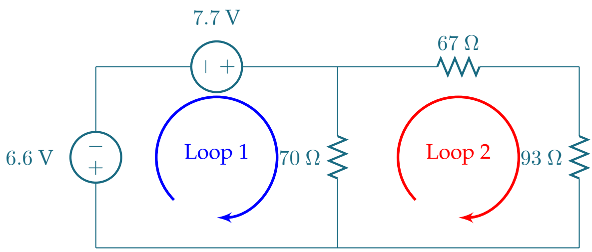
1. How to identify the loops in an electric circuit.
2. How to define the relationships between mesh currents and the currents through elements in an electric circuit.
3. How to write and solve mesh-current equations for a circuit that contains only independent sources.

Loops and Mesh Currents in an Electric Circuit

Let's begin by considering the circuit shown below:

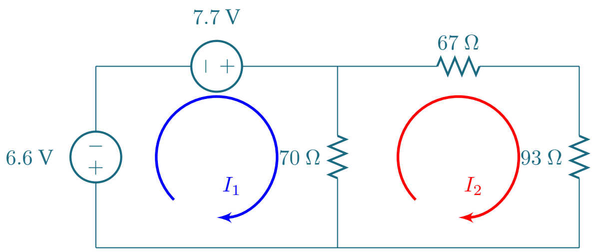


To apply the mesh-current method to this circuit, we first identify loops within the circuit. A loop is simply a closed path through the circuit that originates and ends at the same point. Two possible loops in this circuit are shown below:



Loop 1 traverses clockwise through the 6.6 V source, the 7.7 V source, and the 70 Ω resistor; whereas Loop 2 traverses clockwise through the 70 Ω resistor, the 67 Ω resistor, and the 93 Ω resistor. We might also define a loop through the 6.6 V source, the 7.7 V source, the 67 Ω resistor, and the 93 Ω resistor, but, as we will see below, this loop would be redundant for the purposes of applying the mesh-current method.

To apply the mesh-current method, we introduce the concept of mesh currents, which are simply the currents that we associate with each loop. For our example circuit, for instance, the mesh currents could be defined as I_1 and I_2 as shown below:



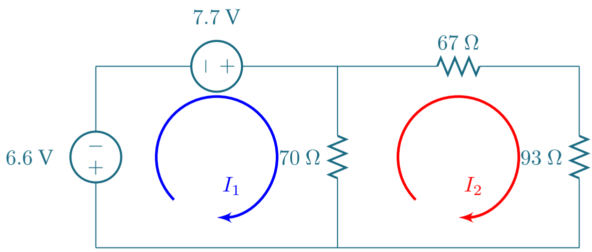
The relationships between these mesh currents and elements currents are as follows:

1. The current flowing upward through the 6.6 V source is equal to I_1.
2. The current flowing left-to-right through the 7.7 V source is equal to I_1.
3. The current flowing left-to-right through the 67 Ω resistor is equal to I_2.
4. The current flowing downward through the 93 Ω resistor is equal to I_2.
5. The current flowing *downward* through the 70 Ω resistor is equal to $I_1 - I_2$.
6. The current flowing *upward* through the 70 Ω resistor is equal to $I_2 - I_1$.

In general, if an element is only part of one loop, then the current flowing through it is equal to that mesh current. If an element is part of two loops, though, then the current flowing through it is determined by the sum or difference of the two mesh currents.

Systematic Equations

To utilize the mesh-current method of analyzing a circuit, we begin by applying Kirchhoff's voltage law around each of the loops. Beginning at loop 1,



we move around the loop in a clockwise direction, add all of the voltage drops, then set the sum equal to zero:

$$6.6 - 7.7 + 70(I_1 - I_2) = 0. \qquad \text{(Loop 1)}$$

Moving upward through the 6.6 V source results in a voltage drop equal to 6.6, moving left to right through the 7.7 V source results in a voltage drop equal to -7.7 (a voltage *gain* equal to 7.7), and moving downward through the 70 Ω resistor results in a voltage drop equal to the resistance (70) times the downward current ($I_1 - I_2$). Next, using the same methodology while moving around loop 2 in a clockwise direction we get the equation:

$$70(I_2 - I_1) + 67I_2 + 93I_2 = 0. \qquad \text{(Loop 2)}$$

If we group common terms in these equations, we can obtain the following system of equations:

$$70I_1 - 70I_2 = 1.1, \qquad \text{(Loop 1)}$$

and

$$-70I_1 + (70 + 67 + 93)I_2 = 0. \qquad \text{(Loop 2)}$$

If we solve this system of equations, we can determine that the mesh currents are equal to

$$I_1 = 22.6 \text{ mA},$$

and

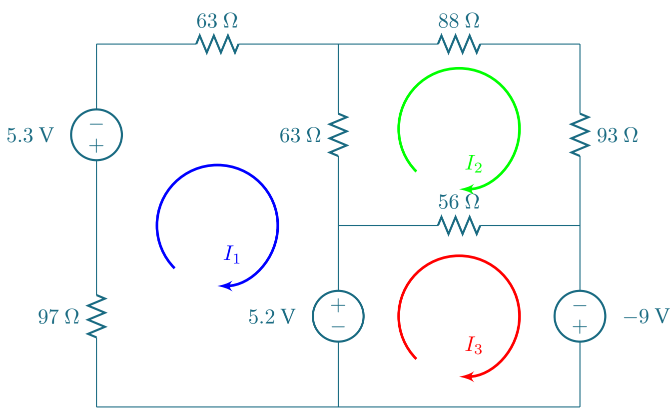
$$I_2 = 6.9 \text{ mA}.$$

If, for example, we wanted to determine the voltage drop downward across the $70\ \Omega$ resistor, we could first note that the downward current through this resistor is $I_1 - I_2 = 15.7$ mA, so that the voltage drop is

$$V_{70\ \Omega} = 70(0.0157) = 1.1 \text{ V}.$$

Of course we could have determined this directly by summing the two voltage sources that are connected across this resistor, but the mesh-current method is a systematic approach that will work for many situations in which determining voltages and currents is not so easily done.

As another example, let's use the mesh-current method to analyze this circuit for which we've associated mesh currents with three loops:



The mesh equation for loop 1 is

$$97I_1 + 5.3 + 63I_1 + 63(I_1 - I_2) + 5.2 = 0, \qquad \text{(Loop 1)}$$

the equation for loop 2 is

$$63(I_2 - I_1) + 88I_2 + 93I_2 + 56(I_2 - I_3) = 0, \qquad \text{(Loop 1)}$$

and the equation for loop 3 is

$$-5.2 + 56(I_3 - I_2) - (-9) = 0. \qquad \text{(Loop 3)}$$

Regrouping the common terms, we have the following system of equations:

$$(97 + 63 + 63)I_1 - 63I_2 + 0I_3 = -5.2 - 5.3 \qquad \text{(Loop 1)}$$

$$-63I_1 + (63 + 88 + 93 + 56)I_2 - 56I_3 = 0 \qquad \text{(Loop 2)}$$

$$0I_1 - 56I_2 + 56I_3 = 5.2 - 9 \qquad \text{(Loop 3)}$$

In matrix-vector notation, we can write this system of equations as

$$\begin{bmatrix} 223 & -63 & 0 \\ -63 & 300 & -56 \\ 0 & -56 & 56 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -10.5 \\ 0 \\ -3.8 \end{bmatrix}.$$

The solution to this system of equations is

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -0.0555 \\ -0.0299 \\ -0.0978 \end{bmatrix}.$$

Therefore, if we need to determine the power associated with the 5.2 V source, we could first determine the current flowing upward through the source as

$$I_{5.2\text{ V}} = I_3 - I_1 = -0.0422 \text{ A},$$

so that the power associated with this source is

$$P_{5.2\,\mathrm{V}} = -(5.2)(-0.0422) \simeq 219\,\mathrm{mW}.$$

Note that, because this power is positive, this source is absorbing power from the other sources. This is the type of thing that happens when we design a circuit to charge a battery.