
Node-Voltage Method: Part 1

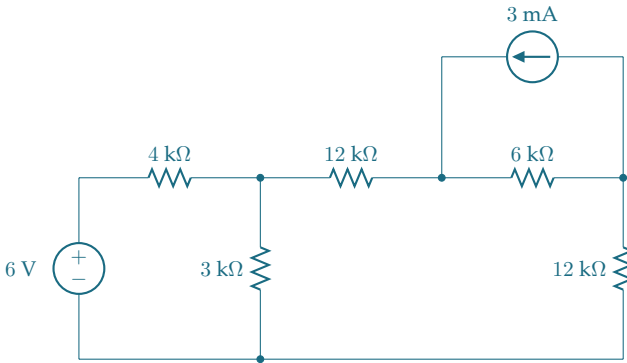
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This lesson provides an introduction to the node-voltage method as it is commonly used in the study of electric circuits. When you complete this lesson, you should know the following:

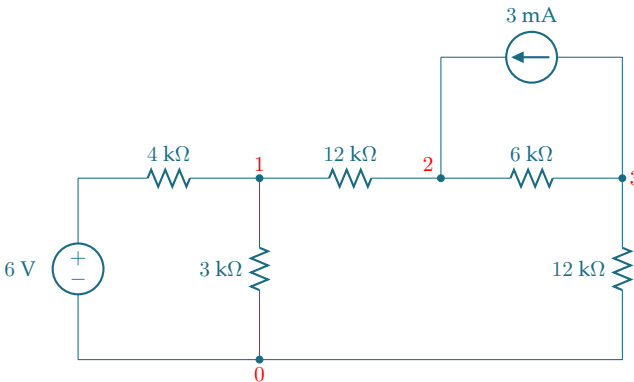
1. How to identify the nodes in an electric circuit.
2. How to write and solve node-voltage equations for a circuit that contains only independent sources.

Nodes in an Electric Circuit

Let's begin by considering the circuit shown below:

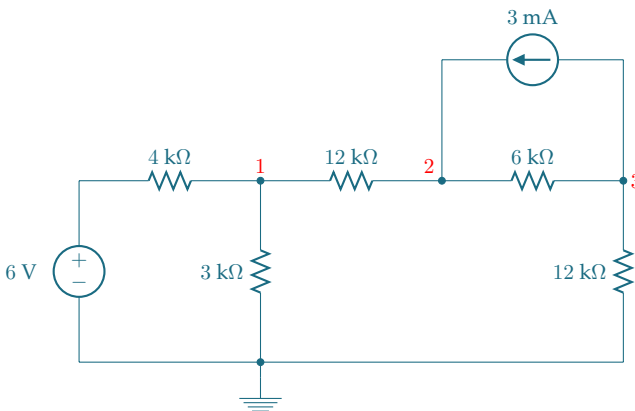


Although a node is usually defined as a connection between two or more circuit elements, when applying the node-voltage method we typically think of a node as a connection between three or more elements. For the circuit shown above, for instance, we would identify the four nodes that are numbered (0 through 3) in the following figure:



Node Voltages

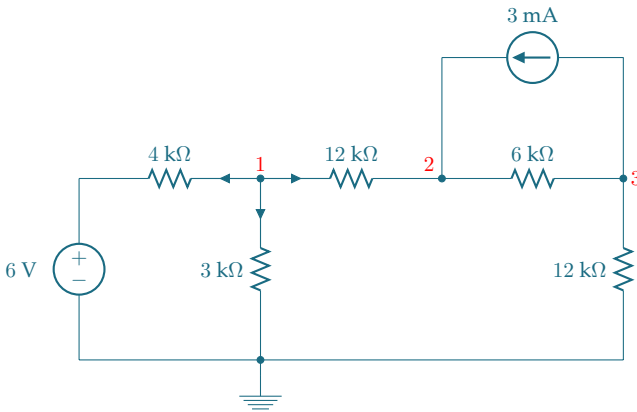
The purpose of the node voltage method is to find the relative voltages at all of the circuit's nodes. Because we are only interested in the relative voltages, we can arbitrarily select one of the nodes as our *reference node*, and assign to it a relative voltage equal to 0 V. If, for instance, we select node 0 as the reference node for our example circuit, then we might denote this as shown below where the *ground symbol* attached to the bottom node is used to indicate that we have assigned that node a relative voltage of 0 V:



The voltage at node 1, then, is denoted as V_1 ; the voltage at node 2 is denoted as V_2 ; and the voltage at node 3 is denoted as V_3 . Once we have determined the voltages at these nodes we will can use the basic principles of circuit analysis to determine the voltages across, currents through, and powers associated with all of the circuit elements. The voltage drop across the $3\text{ k}\Omega$ resistor (from top to bottom), for instance, would be determined as $V_1 - 0$; the voltage drop across the $4\text{ k}\Omega$ resistor (from right to left) would be determined as $V_1 - 6$; and the current through the $6\text{ k}\Omega$ resistor (flowing from left to right) would be determined as $(V_2 - V_3)/6000$.

Systematic Equations

To utilize the node-voltage method for analyzing a circuit, we begin by applying Kirchhoff's current law at each of the non-reference nodes. Beginning at node 1,

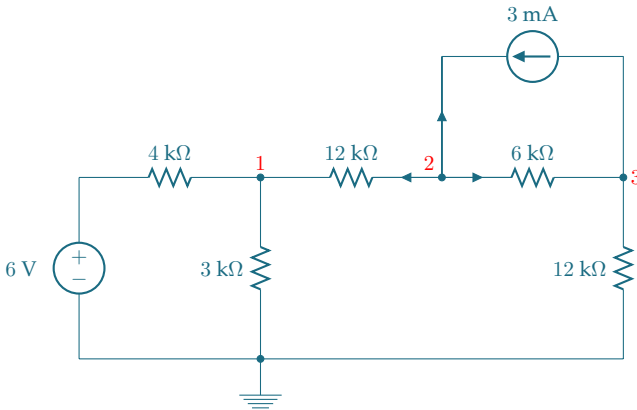


we add all the currents flowing out of the node and set the sum equal to zero:

$$\frac{V_1 - 6}{4000} + \frac{V_1 - V_2}{12000} + \frac{V_1 - 0}{3000} = 0. \quad (V_1)$$

Notice how we use Ohm's Law to express the currents flowing out of the node as functions of the node voltages and the resistances. For the current flowing through the 4 kΩ resistor, we need to recognize that the voltage to the left of the resistor is equal to 6 V because of the voltage source between that point and the reference node (or ground).

Next we move to node 2,

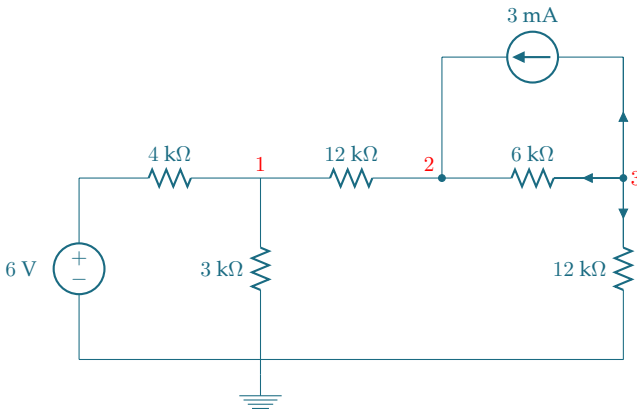


where we again sum all the currents flowing out of the node and set the result equal to zero:

$$\frac{V_2 - V_1}{12000} - \frac{3}{1000} + \frac{V_2 - V_3}{6000} = 0. \quad (V_2)$$

Note that the current flowing upward from the node (and through the current source) is equal to *negative* 3 mA (or $-3/1000$) because the current source is providing a current with a reference direction that flows *into* the node.

Finishing with node 3,



we add all the currents flowing out of the node and set that sum equal to zero:

$$\frac{V_3 - V_2}{6000} + \frac{3}{1000} + \frac{V_3 - 0}{12000} = 0. \quad (V_3)$$

Note that this time the current flowing upward from the node (and through the current source) is equal to *positive* 3 mA (or 3/1000) because the current source is providing a current with a reference direction that flows *out* of the node.

Now we can regroup these three equations as

$$\frac{8}{12000}V_1 - \frac{1}{12000}V_2 + 0V_3 = \frac{6}{4000}, \quad (V_1)$$

$$\frac{-1}{12000}V_1 + \frac{3}{12000}V_2 - \frac{1}{6000}V_3 = \frac{3}{1000}, \quad (V_2)$$

and

$$0V_1 - \frac{1}{6000}V_2 + \frac{3}{12000}V_3 = \frac{-3}{1000}. \quad (V_3)$$

If we multiply each of these equations by 12000 they can be rewritten as

$$8V_1 - V_2 + 0V_3 = 18, \quad (V_1)$$

$$-V_1 + 3V_2 - 2V_3 = 36, \quad (V_2)$$

$$0V_1 - 2V_2 + 3V_3 = -36, \quad (V_3)$$

and we can then express this system of equations in matrix-vector form:

$$\begin{bmatrix} 8 & -1 & 0 \\ -1 & 3 & -2 \\ 0 & -2 & 3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 18 \\ 36 \\ -36 \end{bmatrix}$$

At this point we can use **Cramer's rule**, a programmable calculator, or a programming language like Matlab to solve for the un-

known node voltages. Here, for example, is an example of how an interactive session in Matlab might be used to solve for the voltages:

```
>> A = [8 -1 0; -1 3 -2; 0 -2 3];
>> y = [18; 36; -36];
>> v = inv(A)*y
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v =

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3.4054
9.2432
-5.8378
```

Therefore, we would conclude that

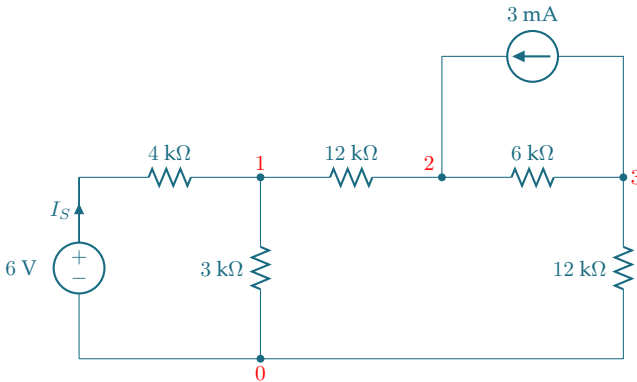
$$V_1 = 3.405 \text{ V},$$

$$V_2 = 9.243 \text{ V},$$

and

$$V_3 = -5.838 \text{ V}.$$

Suppose, then, that we wanted to know the power provided by the 6 V source:



First, we would note that the current through the 6 V source, I_S , is the same as the current through the 4 k Ω resistor, and that current

can be evaluated as

$$I_S = \frac{6 - V_1}{4000} = \frac{6 - 3.405}{4000} = 0.6488 \text{ mA.}$$

Therefore, the power supplied by the voltage source is

$$P_S = -6I_S = -3.893 \text{ mW,}$$

where the negative sign reminds us that power is provided by the source.