The Current Division Method

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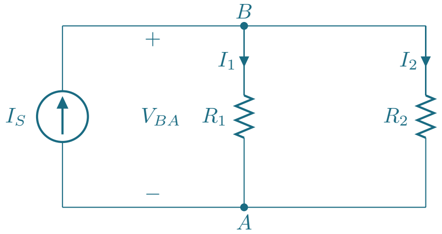
This lesson provides an overview of the current division method as it is commonly used in the study of electric circuits. When you complete this lesson, you should know the following:

1. How to apply the current division method to determine the current that flows through two resistors that are connected in parallel.
2. How to recognize when you can and cannot use the current division method to solve for currents in an electric circuit.

Current Division

Let's begin by considering the circuit shown below:



Because the two resistors are in parallel between the nodes B and A, the equivalent resistance between those nodes is

$$R_{BA} = \frac{R_1 R_2}{R_1 + R_2},$$

and the voltage drop from node B to node A is

$$\begin{aligned} V_{BA} &= I_S R_{BA} \\ &= I_S \frac{R_1 R_2}{R_1 + R_2}. \end{aligned}$$

The current through the resistor R_1, then, is

$$\begin{aligned} I_1 &= \frac{V_{BA}}{R_1} \\ &= I_S \frac{R_2}{R_1 + R_2}. \end{aligned}$$

Similarly, the current through the resistor R_2 is

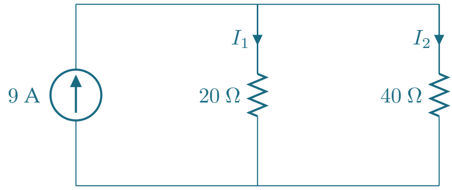
$$\begin{aligned} I_2 &= \frac{V_{BA}}{R_2} \\ &= I_S \frac{R_1}{R_1 + R_2}. \end{aligned}$$

According to these relationships:

When two resistors are connected in parallel, the portion of the current that flows through one of the resistors is determined by the ratio of the other resistor's resistance to the sum of the two resistances.

If, for instance, $R_1 = 100\ \Omega$ and $R_2 = 200\ \Omega$, then two-thirds of the current will flow through R_1, and one-third of the current will flow through R_2.

Example 1. *Consider a simple circuit with two resistors and a current source:*



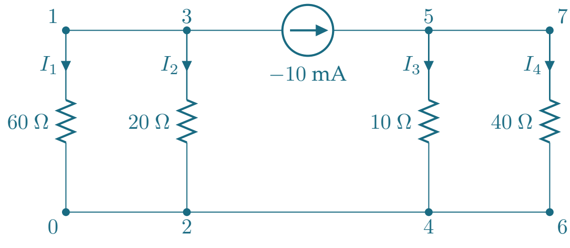
The current through the 20 Ω *resistor is:*

$$I_1 = 9\frac{40}{20+40} = 6\ A,$$

and the current through the 40 Ω *resistor is:*

$$I_2 = 9\frac{20}{20+40} = 3\ A.$$

Example 2. *Let's use the voltage division method to solve for all the currents and voltages in this circuit:*



First, note that the current source provides a current of $-10\,\mathrm{m}A$ *from node 3 to node 5, and a current of* $+10\,\mathrm{m}A$ *from node 5 to node 3. Using the current division method, then, the current* (I_1) *from node 1 to node 0 is equal to*

$$I_1 = 10\frac{20}{20+60} = 2.5\,\mathrm{m}A,$$

and the current (I_2) *from node 3 to node 2 is equal to*

$$I_2 = 10\frac{60}{20+60} = 7.5\,\mathrm{m}A.$$

Likewise, the current (I_3) *from node 5 to node 4 is equal to*

$$I_3 = -10\frac{40}{10+40} = -8\,\mathrm{m}A,$$

and the current (I_4) *from node 7 to node 6 is equal to*

$$I_4 = -10\frac{10}{10+40} = -2\,\mathrm{m}A.$$

If we assign a relative voltage of $0\ V$ *to nodes 0, 2, 4, and 6, then the voltage at node 1 is*

$$V_1 = 60I_1 = 60(0.0025) = 150\,\mathrm{m}V,$$

which should, of course, be equal to the voltage at node 3:

$$V_3 = 20I_2 = 20(0.0075) = 150\,\mathrm{m}V.$$

The voltage at node 5 is

$$V_5 = 10I_3 = 10(-0.008) = -80\,\mathrm{m}V,$$

which is the same as the voltage at node 7:

$$V_7 = 40I_4 = 40(-0.002) = -80\,\mathrm{m}V.$$

The power supplied by the current source (between nodes 3 and 5) is

$$\begin{aligned} P(3,5) &= [V(3) - V(5)]\, I(3 \to 5) \\ &= [0.150 - (-0.080)](-0.010) \\ &= -2.3\ \mathrm{m}W, \end{aligned}$$

where $I(3 \to 5)$ *is the current flowing from node 3 to node 5, and the negative sign for the power indicates that the power is supplied. The powers absorbed by the resistors are*

$$P(0,1) = I_1^2(60) = (0.0025)^2(60) = 0.375\ \mathrm{m}W,$$

$$P(2,3) = I_2^2(20) = (0.0075)^2(20) = 1.125\ \mathrm{m}W,$$

$$P(4,5) = I_3^2(10) = (0.008)^2(10) = 0.64\ \mathrm{m}W,$$

and

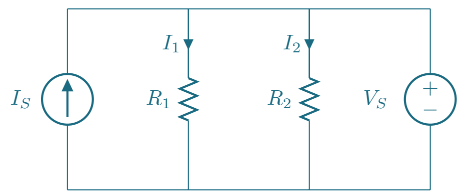
$$P(6,7) = I_4^2(40) = (0.002)^2(40) = 0.16\ \mathrm{m}W.$$

The total power absorbed by the resistors, then, is

$$P_{\text{absorbed}} = (0.375 + 1.125 + 0.64 + 0.16)\ \mathrm{m}W = 2.3\ \mathrm{m}W,$$

so that the total power absorbed is equal to the total power supplied.

Example 3. *The following circuit is an example of a situation for which you cannot apply the current division method:*



To solve for the current through the R_2 *resistor, you* CANNOT *use* *current division:*

$$I_1 = I_S \frac{R_1}{R_1 + R_2}.$$

Because the voltage source is connected in parallel with the resistors, the current division method cannot be applied. You can, however, simply apply Ohm's Law to solve for the currents:

$$I_1 = \frac{V_S}{R_1},$$

and

$$I_2 = \frac{V_S}{R_2}.$$