# The Voltage Division Method

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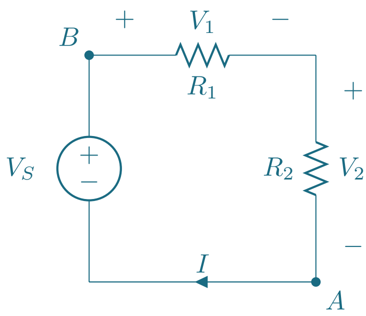
This lesson provides an overview of the voltage division method as it is commonly used in the study of electric circuits. When you complete this lesson, you should know the following:

1. How to apply the voltage division method to determine the voltages across two resistors that are connected in series.
2. How to recognize when you can and cannot use the voltage division method to solve for voltages in an electric circuit.

# Voltage Division

Let's begin by considering the circuit shown below:




The voltage drop across the two resistors is equal to the voltage drop from node $B$ to node $A$, which is equal to the source voltage:

$$V_{BA} = V_S.$$

Because the two resistors are in series, the relationship between the current through them and the voltage across them is

$$I = \frac{V_{BA}}{R_1 + R_2}.$$

By using Ohm's Law, then, we can determine the voltage drop across each resistor:

$$\begin{aligned} V_1 &= I R_1 \\ &= \frac{V_{BA}}{R_1 + R_2} R_1 \\ &= V_{BA} \frac{R_1}{R_1 + R_2}, \end{aligned}$$

and

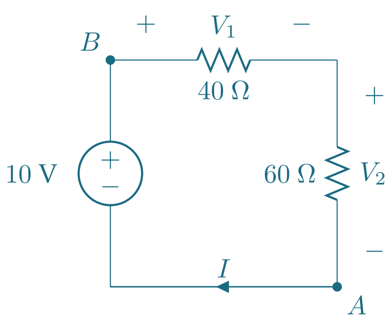
$$\begin{aligned} V_2 &= IR_2 \\ &= \frac{V_{BA}}{R_1 + R_2} R_2 \\ &= V_{BA} \frac{R_2}{R_1 + R_2}. \end{aligned}$$

According to these relationships:

> When two resistors are connected in series, the portion of the voltage that drops across one of the resistors is determined by the ratio of its resistance to the sum of the two resistances.

If, for instance, $R_1 = 100\ \Omega$ and $R_2 = 200\ \Omega$, then one-third of the voltage will drop across $R_1$, and two-thirds of the voltage will drop across $R_2$.

**Example 1.** *Consider a simple circuit with two resistors and a voltage source:*



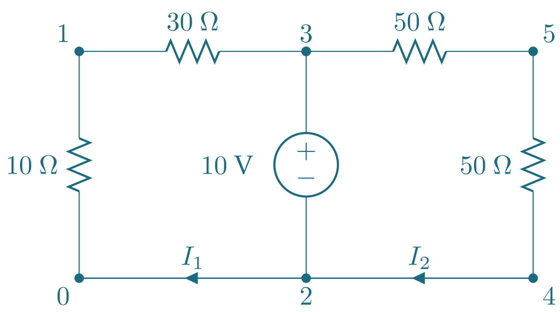

*The voltage across the* $40\ \Omega$ *resistor is:*

$$V_1 = 10 \frac{40}{40 + 60} = 4\ V,$$

*and the voltage across the* 60 Ω *resistor is:*

$$V_2 = 10\frac{60}{40+60} = 6\ V.$$

**Example 2.** *Let's use the voltage division method to solve for all the currents and voltages in this circuit:*




*If we assign node* 0 *as our reference, then we have*

$$V_0 = V_2 = V_4 = 0\ V.$$

*Next, the voltage source provides a voltage increase of* 10 *V from node* 2 *to node* 3*, so that*

$$V_3 = 10\ V.$$

*Because we have a voltage drop of* 10 *V across the two resistors on the left of the circuit, the voltage drop from node 1 to node 0 can be determined by the voltage division method:*

$$V_1 = 10\frac{10}{10+30} = 2.5\ V.$$

*Likewise, we have* 10 *V across the two resistors on the right side of the circuit, so the voltage drop from node 5 to node 4 can be determined as*

$$V_5 = 10\frac{50}{50+50} = 5\ V.$$

*Now that we know the voltages at all the nodes, we can use Ohm's Law and Kirchhoff's Current Law to determine all the currents. The current through the two* $50\ \Omega$ *resistors (using the current labeled* $I_2$ *to define the reference direction) is*

$$I_2 = \frac{10}{50+50} = 0.5\,A,$$

*and the current through the* $10\ \Omega$ *and* $30\ \Omega$ *resistors (using the current labeled* $I_1$ *to define the reference direction) is*

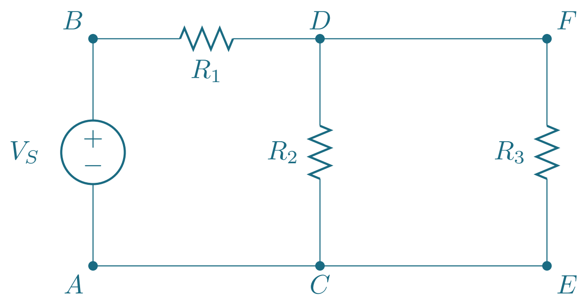
$$I_1 = \frac{-10}{10+30} = -2.5\,A.$$

*Finally, we can use Kirchhoff's Current Law to determine the current through the* $10\ V$ *source as:*

$$I(2 \rightarrow 3) = I_2 - I_1 = 0.5 - (-2.5) = 3\,A,$$

*where the notation* $I(2 \rightarrow 3)$ *is used to specify the current flowing from node 2 to node 3.*

**Example 3.** *The following circuit is an example of a situation for which you need to be careful when using the voltage division method:*




*To solve for the voltage across the* $R_2$ *resistor, you* CANNOT *use the* *voltage division method in this way:*

$$V_{DC} = V_S \frac{R_2}{R_1 + R_2}.$$

*To use voltage division here, we would first need to combine the resistors $R_2$ and $R_3$ as a single equivalent resistor between nodes $C$ and $D$, then use the voltage division method as:*

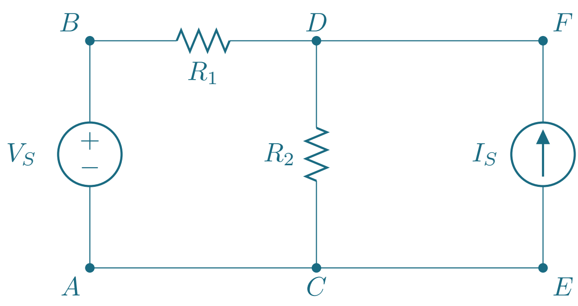
$$V_{DC} = V_S \frac{R_{eq}}{R_1 + R_{eq}},$$

*where*

$$R_{eq} = \frac{R_2 R_3}{R_2 + R_3}.$$

*Remember, you can only use the voltage division method when the two resistors are in series, with no other elements in parallel with either resistor.*

**Example 4.** *Here's another example of a situation for which you cannot use the voltage division method:*




*Because of the current source in parallel with the $R_2$ resistor, you* CANNOT *use the voltage division method in this way:*

$$V_{DC} = V_S \frac{R_2}{R_1 + R_2}.$$

*For this circuit we would need to use some other analysis method to solve for the voltage $V_{DC}$.*