Resistors and Resistance

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This lesson provides an introduction to an important electric-circuit element called the resistor and the equally important concept of resistance. When you complete this lesson, you should know the following:

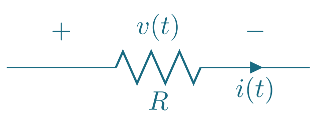
1. How resistors are characterized by a quantity called resistance.
2. Ohm's Law: the mathematical relationship between the current through and voltage across a resistor.
3. The manner in which the electrical power dissipated by a resistor is related to the resistor current and resistor voltage.
4. How to solve for unknown quantities in a simple circuit that has one source and one resistor.

Resistors, Resistance and Ohm's Law

A resistor is a common electric-circuit element that has a very special mathematical relationship between the voltage across and current through its terminals. The schematic symbol we use for a resistor is

where the quantity R is called the *resistance*. Resistance has units called ohms, and we denote this unit with the symbol Ω. If, for instance, a resistor has a resistance equal to $100\ \Omega$, then we might represent the resistor in a circuit like this:

If we label the voltage across and current through a resistor like this



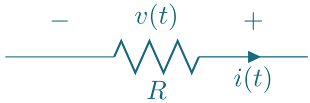
then the voltage and current are related through a mathematical relationship called *Ohm's Law*:

$$v(t) = Ri(t).$$

That is, the voltage is proportional to the current, and the constant of proportionality is equal to the resistance. Because of this relationship, the resistance must have units equal to volts per amp, so an ohm is equal to one volt divided by one amp:

$$\Omega = \frac{\text{V}}{\text{A}}.$$

When applying Ohm's Law, it is important to pay careful attention to the voltage polarity and current direction. If, for instance, the voltage and current are labeled like this:



then the voltage and current would be related by the following relationship:

$$v(t) = -Ri(t).$$

> It is extremely important that you take the time to understand the role of voltage polarity and current direction when applying Ohm's Law.

Power Dissipation in Resistors

Because the power we associate with an electric-circuit element is the product of the voltage across and current through the element, we can use Ohm's Law to determine the power we associate with a resistor as:

$$\begin{aligned} p(t) &= v(t)i(t) \\ &= Ri^2(t). \end{aligned}$$

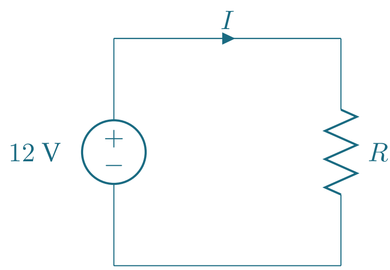
The resistor's power, then, is proportional to the square of the current through the device. Likewise, we could use Ohm's Law to replace the current:

$$\begin{aligned} p(t) &= v(t)i(t) \\ &= v^2(t)/R. \end{aligned}$$

Therefore, we can use the preceding relationships to determine a resistor's power from either the current through or voltage across the resistor. Notice that the power will always be a positive quantity, so resistors are always sinks for electric power.

Some Examples

Here's a simple circuit with one resistor and one voltage source:



Because the positive and negative terminals of the voltage source are connected directly across the resistor, the voltage across the resistor is 12 V. The direction for the current that is labeled I is such that it enters the resistor on the side with the positive polarity and exits on the side with the negative polarity. Accordingly, we can use Ohm's Law to relate the voltage and current as:

$$V = RI.$$

If, for instance, we wanted to design a resistor to provide 2 A of current, then we would specify the resistance to be:

$$R = \frac{V}{I} = \frac{12}{2} = 6\ \Omega.$$

If, on the other hand, we wanted to determine the current when the resistance was equal to 1 kΩ, then

$$I = \frac{V}{R} = \frac{12}{1000} = 0.012\ \text{A} = 12\ \text{mA}.$$

Using the same circuit, suppose we knew that resistor dissipated 8 W of power. In this case, we could determine the value of the resistance according to the following relationship:

$$P = V^2/R \Longrightarrow R = \frac{V^2}{P} = \frac{12^2}{8} = 18\ \Omega.$$