Equivalent Resistance

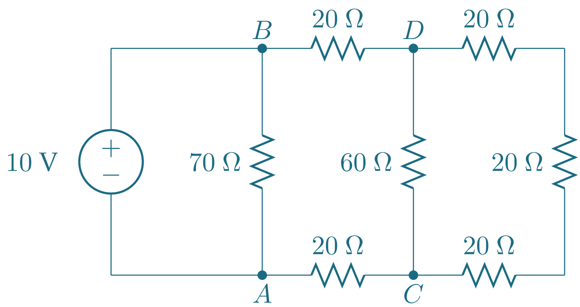
Timothy J. Schulz

This lesson provides an overview of the concept of equivalent resistance as it is commonly used in the study of electric circuits. When you complete this lesson, you should know the following:

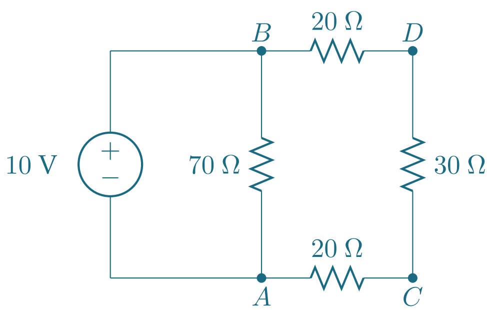
1. The meaning of equivalent resistance for a network of resistors.
2. The method for determining the equivalent resistance for two or more resistors in series.
3. The method for determining the equivalent resistance for two or more resistors in parallel.
4. The systematic method for determining the equivalent resistance for several resistors in a network.

Equivalent Resistance

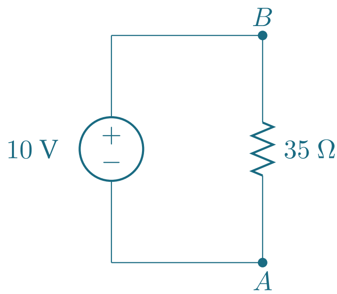
Let's begin by considering the circuit shown below:



For the purpose of analyzing the currents and voltages to the left of nodes C and D, we could replace the circuit with this one:



Here, we have replaced the three 20 Ω resistors and the 60 Ω resistors with a single 30 Ω resistor. By selecting this particular resistance, all the currents and voltages to the left of nodes C and D are the same for both of these circuits. Furthermore, if we were only interested in the currents and voltages to the left of nodes A and B, then we could replace the circuit with this one:



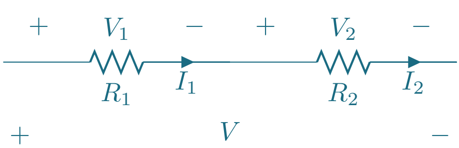
Here, we have replaced the two 20 Ω resistors, the 30 Ω resistor, and the 70 Ω resistor with an equivalent resistance of 35 Ω. Using this simple circuit, we could determine that the current flowing from the 10 V source is

$$I = \frac{10}{35} = 285.67 \text{ mA}.$$

To make this simplification for other circuits, you will need to master two concepts: 1) equivalent resistance for resistors in series; and 2) equivalent resistance for resistors in parallel.

Resistors in Series

Let's take a look at the following circuit segment:



From Ohm's Law we know that

$$V_1 = I_1 R_1 \quad \text{and} \quad V_2 = I_2 R_2,$$

from Kirchhoff's Current Law we know that

$$I_1 = I_2,$$

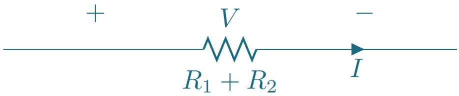
and from Kirchhoff's Voltage Law we know that

$$V = V_1 + V_2.$$

If we put all this together, then we can see that

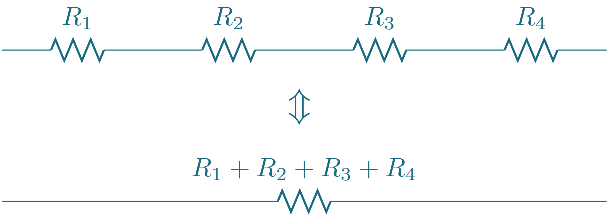
$$\begin{aligned} V &= I_1 R_1 + I_2 R_2 \\ &= I(R_1 + R_2), \end{aligned}$$

where $I = I_1 = I_2$. Therefore, the series combination for the two resistors provides the same relationship between the voltage drop and current current flow as would a single resistor of resistance $R = R_1 + R_2$:



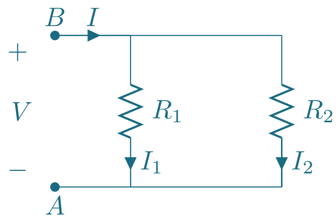
This result—that the equivalent resistance for two resistors in series is equal to the sum of the individual resistances—also applies for three, four, or any number of resistors in series:

> Two or more resistors that are connected in series are equivalent to a single resistor with a resistance equal to the sum of their individual resistances.



Resistors in Parallel

Now, let's take a look at a circuit segment with resistors connected in parallel:



From Ohm's Law we know that

$$I_1 = \frac{V}{R_1} \quad \text{and} \quad I_2 = \frac{V}{R_2},$$

and from Kirchoff's Current Law we know that

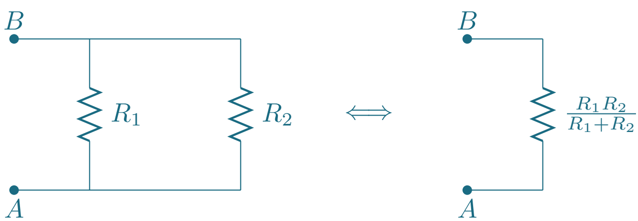
$$\begin{aligned} I &= I_1 + I_2 \\ &= \frac{V}{R_1} + \frac{V}{R_2} \\ &= V\left(\frac{1}{R_1} + \frac{1}{R_2}\right). \end{aligned}$$

From this relationship, then, we can infer that the two resistors in parallel are equivalent to a single resistor with a resistance that satisfies the relationship:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2},$$

or

$$R = \frac{R_1 R_2}{R_1 + R_2}.$$



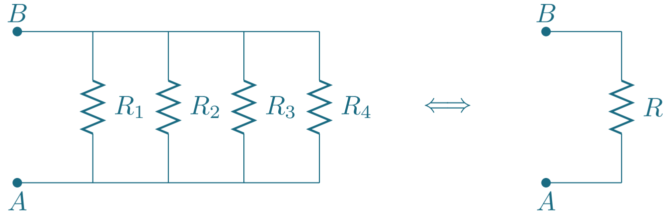
The reciprocal of resistance is called *conductance*, and is usually denoted by the letter G:

$$G = \frac{1}{R}.$$

The unit for conductance is the reciprocal of ohms, which is called siemens and assigned the symbol S. Using this definition for conductance, we can describe a general result for resistors that are connected in parallel:

> Two or more resistors that are connected in parallel are equivalent to a single resistor with a *conductance* equal to the sum of their individual *conductances*.

Four resistors in parallel, for instance, can be replaced by a single equivalent resistance:



where

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}.$$

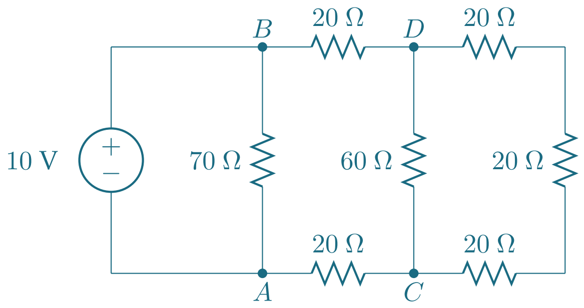
It is useful to note that the equivalent resistance for two resistors in parallel is

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2},$$

but for three or more resistors in parallel, it is best to use the general relationship for adding conductances.

Example

Let's return to our first example:

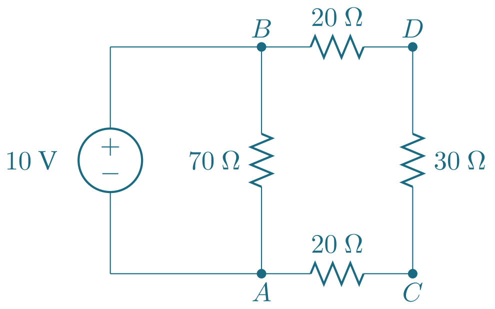


Note that the three resistors on the right side of the circuit can be combined in series to form an equivalent resistance equal to $20 + 20 + 20 = 60\ \Omega$. That equivalent resistor is in parallel with the $60\ \Omega$ resistor between nodes C and D, so, if we combine those we can replace the four resistors with an equivalent resistance determined by

$$\frac{1}{R_{CD}} = \frac{1}{60} + \frac{1}{20+20+20} = \frac{1}{60} + \frac{1}{60} = \frac{1}{30}.$$

Therefore, the equivalent resistance between nodes C and D is

$$R_{CD} = 30\ \Omega.$$



Now we can combine the three resistors on the right of the new circuit to form an equivalent resistance equal to $20 + 30 + 20 = 70\ \Omega$. This equivalent resistor is in parallel with the $70\ \Omega$ resistor between nodes A and B, so, if we combine those we can replace the four resistors with an equivalent resistance determined by

$$\frac{1}{R_{AB}} = \frac{1}{70} + \frac{1}{20 + 30 + 20} = \frac{1}{70} + \frac{1}{70} = \frac{1}{35}.$$

Therefore, the equivalent resistance between nodes A and B is

$$R_{AB} = 35\ \Omega.$$