Power and Energy in Electric Circuits

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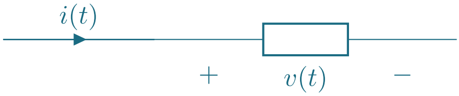
This lesson provides an overview of power and energy as they apply to the study of electric circuits. When you complete this lesson, you should know the following:

1. The definition and basic meaning of electric power.
2. The mathematical relationship between electric current, voltage, and power.
3. The meaning and importance of current direction and voltage polarity for determining the sign (positive or negative) of the power for a device in an electric circuit.
4. The difference between a power source and a power sink.
5. The definition and basic meaning of electric energy.
6. The mathematical relationship between electric power and energy.

Electric Power

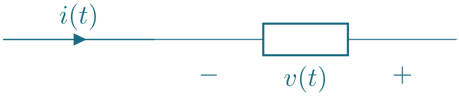
The electric power we associate with an electric device is the product of the current through the device with the voltage across the device. The unit for power is the watt (W), where one watt is a volt (V) times an amp (A). When computing power, it is important to use the appropriate sign conventions. If, for instance, we define our current and voltage like this:



so that the current arrow enters the device on the side of the positive polarity reference, then the power $p(t)$ will be

$$p(t) = v(t)i(t). \tag{1}$$

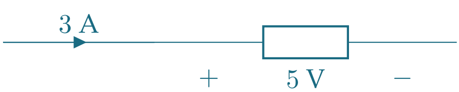
If, however, we define our current and voltage like this:

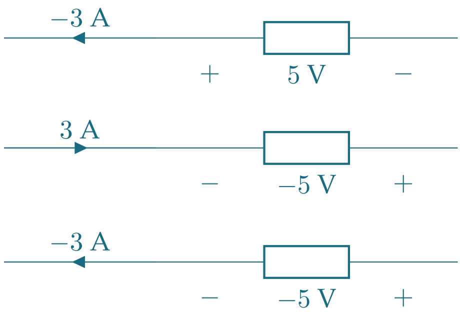


so that the current arrow enters the device on the side of the negative polarity reference, then the power will be

$$p(t) = -v(t)i(t). \tag{2}$$

Being careless or sloppy with this convention is one of the most common mistakes students make when they are learning the principles of electric circuits. Because of this, it is worthwhile to take a few minutes to verify that the power for each of the following elements is equal to $+15W$:





When a power quantity is a positive number, then we say that the device is absorbing power. This will happen, for instance, with a light bulb. When a power quantity is negative, we say that the device is supplying power to a circuit. This will happen with a battery.

Devices that absorb power (positive number for power) are called *power sinks*. Devices that supply power (negative number for power) are called *power sources*.

Because the current and voltage associated with a device in an electric circuit can, in general, fluctuate with time, it is possible for a device to absorb power at some times and to supply power at others. Consider, for example, a device with the following current and voltage:

The power we associate with this device is:

$$\begin{aligned} p(t) &= -\cos(\pi t)\sin(\pi t) \\ &= -\frac{1}{2}\sin(2\pi t). \end{aligned} \tag{3}$$

Therefore, because of the leading negative sign, the device will be a power source when $\sin(2\pi t)$ is positive, and the device will be a power sink when $\sin(2\pi t)$ is negative.

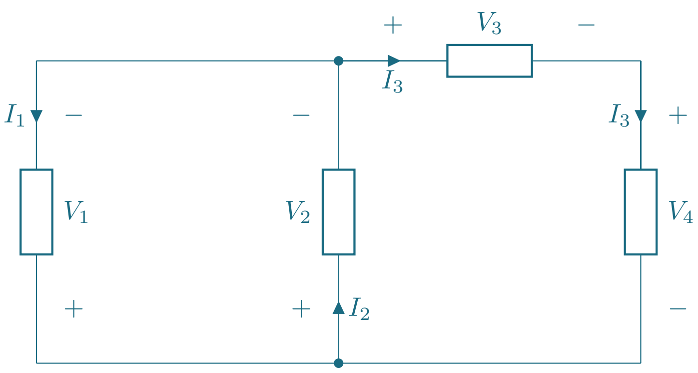
Suppose we had a device that absorbed 6 W of power from a 12 V battery. We could use the definition of power to determine the current the device would draw as:

$$\text{current} = \frac{6\text{ W}}{12\text{ V}} = 0.5\text{ A}.$$

Because the device is a power sink, this current will flow from the battery's positive terminal to its negative.

Conservation of Power

For all of the circuit models we'll examine during our study of electric circuits, the power supplied must be equal to the power absorbed. Because all of the elements that absorb power will have a positive power associated with them, and all of the elements that supply power will have a negative power associated with them, the sum of the power associated with all of the elements in a circuit must be equal to zero. Consider, for example, the following DC circuit:



The power associated with the leftmost device is $P_1 = -I_1V_1$. Moving rightward through the circuit, the powers associated with the other devices are:

$$P_2 = I_2V_2,$$

$$P_3 = I_3 V_3,$$

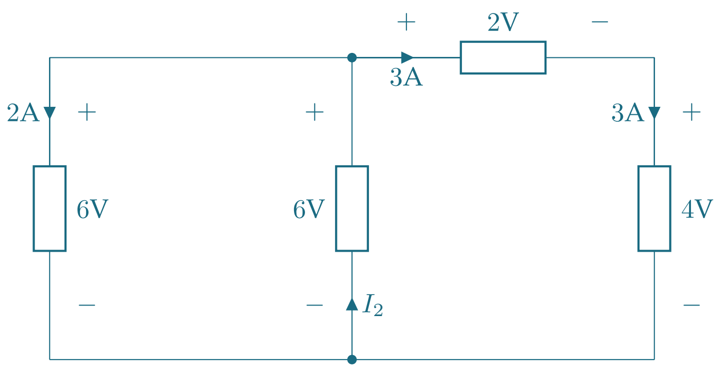
and

$$P_4 = I_3 V_4.$$

Because of conservation of power, we must have the following relationship between these powers:

$$P_1 + P_2 + P_3 + P_4 = -I_1 V_1 + I_2 V_2 + I_3 V_3 + I_3 V_4 = 0.$$

Suppose, for instance, that we know all of the currents and voltages in the following circuit except for the current called I_2:



Because of conservation of power, we must have

$$\begin{aligned} 0 &= (2\text{ A})(6\text{ V}) - (I_2)(6\text{ V}) + (3\text{ A})(2\text{ V}) + (3\text{ A})(4\text{ V}) \\ &= 12\text{ W} - (6\text{ V})I_2 + 6\text{ W} + 12\text{ W} \\ &= 30\text{ W} - (6\text{ V})I_2. \end{aligned} \quad (4)$$

Therefore,

$$I_2 = \frac{30\text{ W}}{6\text{ V}} = 5\text{ A}. \quad (5)$$

When you are first learning the principles of electric circuits, it is helpful to keep track of the units for all of the quantities in a calculation. This will help you avoid simple mistakes. After you've gained some skill at working with typical circuit equations, though, you'll be able to suppress the units in your calculations.

Electric Energy

The electric energy associated with a device in a circuit is the accumulated power associated with a device over an interval of time. If, for instance, a 60 watt light bulb operates continuously for 2 hours, then its energy consumption is

$$\text{energy} = (60 \text{ watt})(2 \text{ hours}) = 120 \text{ watt} \cdot \text{hours}.$$

The electric energy that is provided to homes and offices is typically measured in thousands of watt·hours, or a unit called the kilo-watt hour (kWh). The lightbulb in the previous example, for instance, would have used 0.12 kWh of energy.

Outside of the electric energy industry, the joule (J) is the standard unit of energy. A joule corresponds to one watt for one second, so the conversion between a kilo-watt hour and a joule is

$$\text{kWh} = (1000 \text{ W})(1 \text{ h}) = (1000 \text{ W})(3600 \text{ s}) = 3.6 \times 10^6 \text{ J}.$$

The general mathematical relationship between power $p(t)$ and the energy over a particular interval of time (t_1, t_2) is

$$\text{energy} = \int_{t_1}^{t_2} p(t)dt. \tag{6}$$

If we define $w(t)$ as the energy over the interval from some initial time $t = t_0$ to the current time t, then we can obtain the power from the energy by:

$$p(t) = \frac{dw(t)}{dt}. \tag{7}$$