Basic Quantities in Electric Circuits

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This lesson provides an overview of the basic physical quantities that are used in the study of electric circuits. When you complete this lesson, you should know the following:

1. The definition and basic meaning of electric charge.
2. The definition and basic meaning of electric current.
3. The definition and basic meaning of electric voltage.
4. The mathematical relationship between electric charge and current.
5. The meaning and importance of current direction and voltage polarity in electric circuits.

Electric Charge

The fundamental element in electric circuits is the *electron*, and the devices and circuits that we will analyze and study are designed to move electrons for the purpose of conveying information or transporting energy.

Begin by thinking about a simple piece of copper wire. The wire will contain many, many electrons, and, if the wire isn't connected to some type of electric circuit, then the electrons will be relatively stationary. If, however, the wire is attached to some electrical devices, then the electrons might start moving in one direction or another. In an electrical communication system, for example, the way those electrons move can be used to send a message from one person to another. In a computer, the way those electrons move can carry out logical or mathematical operations. In your home, the way those electrons move can be converted to light or heat, or they can be used to run a machine or empower a computer.

The number of electrons that are involved in even the most simple situations is enormous. The battery in your car needs to push on the order of 10^{19} electrons per second just to run one of your headlights. You've probably never heard something like that before because engineers don't typically think about electric circuits in terms of the electrons themselves. Instead, they think about the electrical charge associated with those electrons. And rather than measuring and keeping track of the movement of electrons, they keep track of the movement of the electrical charge associated with the electrons.

Electric Current

The charge associated with a single electron is about -1.6×10^{-19} coulombs (the symbol we use for a coulomb is C). Although electrons are the fundamental elements in electric circuits, we are almost always concerned with the rate of movement of the electrons, and we quantify this movement in terms of something we call electric current. When, for example, the rate of movement of electrons through some cross-section of wire corresponds to one coulomb per second, then we say that the electrical current is one ampere (the symbol we use for an ampere is A, and we usually refer to an ampere simply as an amp). Mathematically, we write the relationship between charge and current as:

$$i(t) = \frac{dq(t)}{dt}, \tag{1}$$

where $q(t)$ is the amount of charge that moves though a spatial cross-section of the wire at a particular time. To determine the amount of charge when we know the current, we can use the integral relationship:

$$q(t) = \int_{-\infty}^{t} i(\tau) d\tau, \tag{2}$$

or, when we know the amount of charge at a particular time t_0, we can use this modified relationship:

$$q(t) = q(t_0) + \int_{t_0}^{t} i(\tau) d\tau. \tag{3}$$

When we analyze and design electric circuits, we'll use standard diagrams for the wires and elements in the circuit. We'll use a simple straight line for a wire, and we'll denote the current through a wire with a small arrow head like this.

Because electrons have a negative charge, each electron will contribute a negative current in the direction of its movement. Equivalently, then, there will be a positive current flow in the opposite direction. By a widely accepted convention, electric currents that are induced by the movement of electrons are described according to the equivalent movement of positive charge, which flows in the opposite direction of the electrons. Electrons that move from left to right, for instance, induce a positive current that flows from right to left. Electrons that move from right to left induce a positive current that flows from left to right. If, for example, the flow of electrons from right to left in a wire corresponds to 4 coulombs per second, then we denote this with either of the following notations:

This point is important:

> A positive current flowing from left to right is the same as a negative current flowing from right to left.

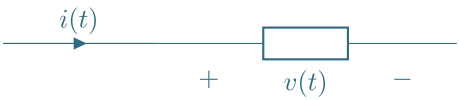
Electric Voltage

When electrons are pushed or pulled through an electrical device—like a resistor, inductor, capacitor, lightbulb, computer, or electric motor—the electrons on one side of the device may have an energy that is larger or smaller than those on the other side. When electrons are pushed through a lightbulb, for instance, some of their energy is used to generate the light, so the electrons on one side will have more energy than the electrons on the other side.

The ratio of energy per coulomb for an electron is called the electric potential, or voltage. The unit for voltage is the volt (V), and one volt is the ratio of one joule (J) per coulomb (C):

$$\text{volt} = \frac{\text{joule}}{\text{coulomb}}. \tag{4}$$

For the generic electrical device denoted by the box in the following circuit segment, the current *through* the device is denoted as $i(t)$ and the voltage *across* the device is denoted as $v(t)$.



It is helpful to always think of the current as flowing *through* a device and the voltage as changing *across* the device.

The direction of the current through the device and the polarity of the voltage drop across the device are extremely important. For the previous illustration, the current flows from left to right, and the voltage *drops* from left to right. If, for instance, the value for the voltage is $v(t) = 4$ V, then we would say that the voltage drops by positive 4 volts from left to right across the element. If, however, the voltage is $v(t) = -4$ V, then we would say that the voltage drops by negative 4 volts (or increases by 4 volts) from left to right, or we would say that the voltage drops by positive 4 volts from right to left.

The relationship between current and voltage for a particular electrical device is determined by the device's physical characteristics. For devices called *resistors*, the relationship is simple:

$$v(t) = Ri(t), \tag{5}$$

where R is a quantity called the *resistance*. For devices called *capacitors*, the relationship is

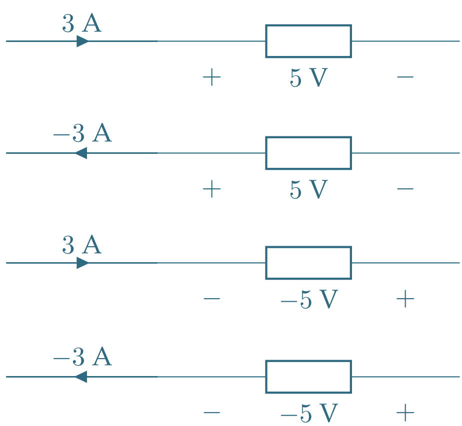
$$i(t) = C\frac{dv(t)}{dt}, \tag{6}$$

where C is a quantity called the *capacitance*. For devices called *inductors*, the relationship is

$$v(t) = L\frac{di(t)}{dt}, \tag{7}$$

where L is a quantity called the *inductance*. We'll learn more about each of these devices in subsequent lessons, but, for now, it is important to understand that each of these devices has a clear mathematical relationship between the current through and voltage across the device. Because of this, if we know one of the quantities (voltage or current), then we can compute the other (current or voltage).

It is worthwhile to spend time gaining familiarity with the concept of voltage polarity and current direction. Take a few minutes to study the following diagrams and convince yourself that they all represent the same circuit segment:



DC vs AC Circuits

When the voltages and currents in a circuit are constant and do not change with time, such as $v(t) = 4$ or $i(t) = -3$, we classify the circuit as a DC or *direct current* circuit. When the voltages and currents in a circuit change with time, such as $v(t) = -\cos(\pi t)$ or $i(t) = e^{-t}$, we classify the circuit as an AC or *alternating current* circuit.

When we analyze or design DC circuits, it is common—but not necessary—to drop the dependence on time and to denote the currents and voltages with upper case symbols such as I and V.